ECE HW #3

SOLNS!

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#1 (a) $y(t) = \int_{-\infty}^{t/2} x(\tau)\, d\tau$

1) Linearity: $a_1 x_1(t) + a_2 x_2(t) \rightarrow \int_{-\infty}^{t/2} (a_1 x_1(\tau) + a_2 x_2(\tau))\, d\tau$

$$= a_1 \int_{-\infty}^{t/2} x_1(\tau)\, d\tau + a_2 \int_{-\infty}^{t/2} x_2(\tau)\, d\tau$$

$$= a_1 y_1(t) + a_2 y_2(t)$$

Linear

2) Time Invariance: $x(t - t_0) \xrightarrow{T} \int_{-\infty}^{t/2} x(\tau - t_0)\, d\tau$ $\qquad \tau' = \tau - t_0$

$$\int_{-\infty}^{t/2 - t_0} x(\tau')\, d\tau'$$

$$x(t) \xrightarrow{T} \int_{-\infty}^{t/2} x(\tau)\, d\tau \xrightarrow[t_0]{\text{Delay}} \int_{-\infty}^{\frac{t-t_0}{2}} x(\tau)\, d\tau \neq y(t/2 - t_0)$$

Time varying

3) Instantaneous: No, since $y(t)$ depends on past input.

4) Causality: Yes

5) Invertible: $\frac{dy(t)}{dt} = \frac{1}{2}x\left(\frac{t}{2}\right)$

$x\left(\frac{t}{2}\right) = 2\frac{dy(t)}{dt}$ $\rightarrow$ invertible

6) Stability: if $|x(t)| \leq M \rightarrow$

$|y(t)| = \left|\int_{-\infty}^{t/2} x(\tau)d\tau\right| \leq \int_{-\infty}^{t/2} |x(\tau)|\, d\tau \leq \int_{-\infty}^{t/2} M\, d\tau \rightarrow \infty$

as $t \rightarrow \infty$

not stable.

(b) $y(t) = 3x(3t+3)$

1) Linearity: $a_1x_1(t) + a_2x_2(t) \xrightarrow{T} 3\left(a_1x_1(3t+3) + a_2x_2(3t+3)\right)$

$= 3a_1x_1(3t+3) + 3a_2x_2(3t+3)$

$= a_1y_1(t) + a_2y_2(t)$ $\therefore$ Linear

2) Time-Invariance: $x(t) \xrightarrow{t_0} x(t-t_0) \xrightarrow{T} 3x(3t-t_0+3)$

$y(t-t_0) = 3x(3(t-t_0)+3)$ $\neq$

$= 3x(3t-3t_0+3)$

Time varying

3) Instantaneous: No. For example, $y(1) = 3x(6)$

depends on the future

4) Causal: No. (depends on the future input)

5) Invertible $\frac{y(t)}{3} = x(3t+3)$

$\tau = 3t+3 \rightarrow t = \frac{\tau-3}{3}$

$x(\tau) = \frac{1}{3}y\left(\frac{\tau-3}{3}\right)$ $\rightarrow$ Invertible

6) Stability: If $|x(t)| \leq M \rightarrow |y(t)| = |3x(3t+3)|$

$\leq 3M$

$\rightarrow$ stable.

(c) $y(t) = x(t)\, t\, u(t)$

1) Linearity: $a_1 x_1(t) + a_2 x_2(t) \xrightarrow{T}$

$$(a_1 x_1(t) + a_2 x_2(t))\, t\, u(t)$$
$$= (a_1 t x_1(t) + a_2 t x_2(t)) u(t)$$
$$= a_1 y_1(t) + a_2 y_2(t) \quad \text{Linear}$$

2) Time-Invariance: $y(t-t_0) = x(t-t_0)(t-t_0)u(t-t_0)$
$x(t-t_0) \xrightarrow{T} x(t-t_0)\, t\, u(t) \quad \neq$
Time varying

3) Instantaneous: Yes
4) Causality: Yes
5) Invertible: No, since $x(t)$ for $t<0$ is all mapped to zero.
6) Stability: if $|x(t)| \le M \rightarrow |y(t)| = |x(t)\, t\, u(t)|$
$$= |x(t)|\,|t|\,|u(t)|$$
$$\le M|t| \rightarrow \infty$$
unstable.

(d) $y(t) = \frac{d}{dt}\left[e^{-t} x(t)\right]$

1) Linearity: $a_1 x_1(t) + a_2 x_2(t) \xrightarrow{T} \frac{d}{dt}\left[e^{-t}(a_1 x_1(t) + a_2 x_2(t))\right]$

$$= a_1 \frac{d}{dt}\left(e^{-t} x_1(t)\right) + a_2 \frac{d}{dt}\left(e^{-t} x_2(t)\right)$$
$$= a_1 y_1(t) + a_2 y_2(t) \quad \text{Linear}$$

2) Time-Invariance: $y(t-t_0) = \frac{d}{dt}\left(e^{-(t-t_0)} x(t-t_0)\right]$

$x(t-t_0) \xrightarrow{T} \frac{d}{dt}\left[e^{-t} x(t-t_0)\right]$ time-varying

3) Instantaneous: No, derivative depends on the past
4) Causal: Yes
5) Invertible: NO because of integration

(6) Stable: if $|x(t)| \le M$

$$|y(t)| = \left|\frac{d}{dt}\left[e^{-t}x(t)\right]\right|$$

$$\frac{d}{dt}\left(e^{-t}x(t)\right) = -e^{-t}x(t) + e^{-t}\frac{dx(t)}{dt}$$

$$|y(t)| \le \left|-e^{-t}x(t)\right| + \left|e^{-t}\frac{dx(t)}{dt}\right|$$

$$= e^{-t}|x(t)| + e^{-t}\left|\frac{dx(t)}{dt}\right|$$

$$\le Me^{-t} + \underbrace{e^{-t}}_{\infty \text{ as } t \to -\infty}\left|\frac{dx(t)}{dt}\right| \Rightarrow \text{not stable.}$$

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Solutions

(2) a) $\frac{\sin t}{t^2+2}\,\delta(t) = 0$

b) $\frac{\sin\left[\frac{\pi}{2}(t-2)\right]}{t^2+4}\,\delta(1-t) = \frac{\sin(-\pi/2)}{5}\,\delta(1-t)$

$= \frac{-1}{5}\,\delta(1-t)$

c) $\int_{-\infty}^{\infty} \sin(\pi t)\,\delta(2t-3)\,dt = \int_{-\infty}^{\infty} \sin(\pi t)\,\delta\left(2\left(t-\frac{3}{2}\right)\right) dt$

$= \frac{1}{2}\int_{-\infty}^{\infty} \sin(\pi t)\,\delta\left(t-\frac{3}{2}\right) dt$

$= \left(\frac{1}{2}\right)(-1) = -1/2$

d) $\int_{-\infty}^{t-1} e^{-\tau}\,\delta(\tau+2)\,d\tau = \int_{-\infty}^{t-1} e^{-2}\,\delta(\tau+2)\,d\tau$

$= e^{-2}\underbrace{\int_{-\infty}^{t-1} \delta(\tau+2)\,d\tau}$

$= \begin{cases} 1, & \text{if } t-1 \geq -2 \Rightarrow t \geq -1 \\ 0, & \text{if } t-1 < -2, \; t < -1 \end{cases}$

$= e^{-2}\,u(t+1)$

e) $\int_{-\infty}^{\infty} \sin(3(t-1))\,\delta(2t+4)\,dt = \frac{1}{2}\int_{-\infty}^{\infty} \sin(3(t-1))\,\delta(t+2)\,dt$

$= \frac{1}{2}\sin(-9)$

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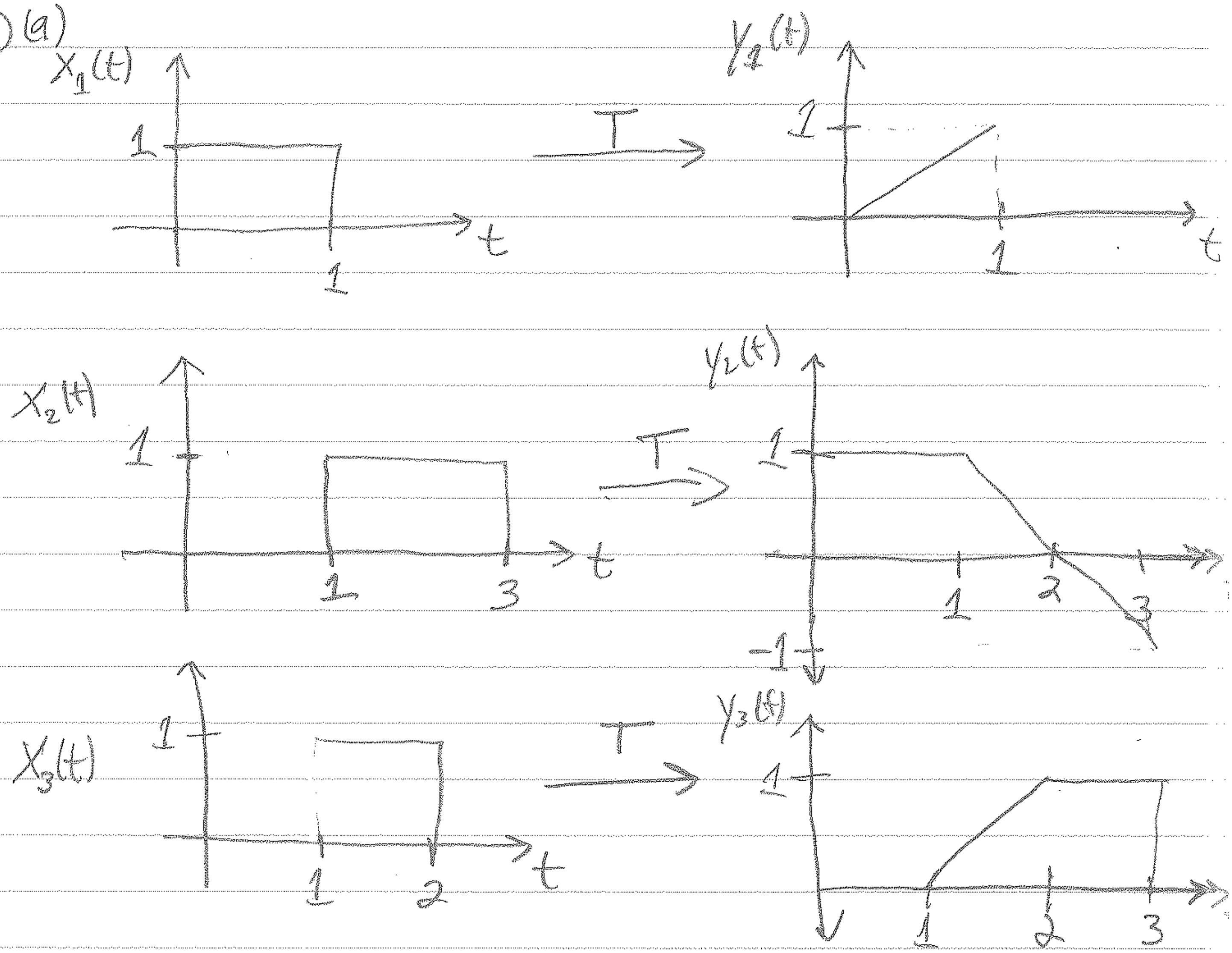
#3. (a)



(b) <u>Not causal</u> — $y_2(t)$ v.s. $y_3(t)$ $\Rightarrow$ the output depends for $0 \le t \le 1$ on whether $x_2(t)$ OR $x_3(t)$ <u>will</u> happen!

(c) <u>Not time invariant!</u> $x_3(t) = x_1(t-1)$, but

$$y_3(t) = (t-1)u(t-1) - (t-2)u(t-2) - u(t-3) \ne \underbrace{(t-1)\left[u(t-1) - u(t-2)\right]}_{=\, y_1(t-1)}$$

(d) <u>Not memoryless!</u> $y_2(t)$ & $y_3(t)$ are different during $0 \le t \le 2$

(e) $x(t) = x_1(t) + 2x_2(t)$, so $y(t) = y_1(t) + 2y_2(t)$ (Linear system)