**ECE 366** Example Problems

- Office Hours: M, W 3:00-4:30 p.m.
- The following questions are from Lathi's book, Second Edition.
- Read Chapter 1.1-1.3 and 1.5.

1. [20] Determine whether the following signals are periodic or not. If periodic, specify the fundamental period and frequency in Hz.

   a) $x(t) = \cos^2(2\pi t)$
   b) $x(t) = \cos(t) + \sin(2\pi t)$
   c) $x(t) = e^{\sin t}$
   d) $x(t) = \cos(4t) + 3e^{-j12t}$
   e) Signal in Figure P1.1-4

2. [20] Determine whether the following signals are energy or power signals. Compute the energy or the power.

   a) $x(t) = \cos(t)u(t)$
   b) $x(t) = 10\cos(5t)\cos(10t)$
   c) $x(t) = \begin{cases} 5\cos(\pi t), -0.5 \le t \le 0.5 \\ 0, otherwise \end{cases}$
   d) $x(t) = te^{-|t|}$
   e) Signal in Figure P1.1-9

3. [12] Determine whether the following signals are even or odd. If neither, find and sketch the even and odd parts of the signals.

   a) $e^{-2t}u(t)$
   b) Signal in Figure P1.5-7
   c) $\sin(\pi t)u(t)$

4. [16] For the signal illustrated in Fig. P1.2-2 sketch:

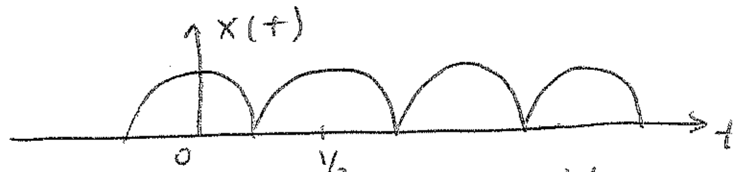
   a) $x(t-4)$
   b) $x(2t-4)$
   c) $\frac{1}{2}x(0.5t+2)-1$
   d) $2x(2-t)$

5. [16] 1.3-6

6. [16] An exponentially damped sinusoidal signal is defined by $x(t) = 20\sin(2000\pi t - \pi / 3)e^{-at}$ , where the exponential parameter $a$ is variable, taking on the set of values $a = 500, 750, 1000$. Using MATLAB, investigate the effect of varying a on the signal $x(t)$ for $-2 \le t \le 2$ milliseconds. Please turn in your m-file and plots of the signal for different values of $a$.

ECE 366 Example Problem solutions

① a) $x(t) = \cos^2(2\pi t)$




periodic with $T_0 = 1/2$

$f_0 = 2\text{ Hz}$

b) $x(t) = \cos(t) + \sin(2\pi t)$

$T_{0,1} = 2\pi$ $\quad T_{02} = 1$

$\frac{T_{0,1}}{T_{0,2}} = 2\pi \rightarrow$ not a rational number

$x(t)$ is aperiodic

c) $x(t) = e^{\sin t}$ , $x(t+T) \stackrel{?}{=} e^{\sin t}$

$e^{\sin(t+T)} \stackrel{?}{=} e^{\sin t}$

if $T_0 = 2\pi k$

$e^{\sin(t+T_0)} = e^{\sin t}$

periodic

$\boxed{T_0 = 2\pi}$

$f_0 = \frac{1}{2\pi}$

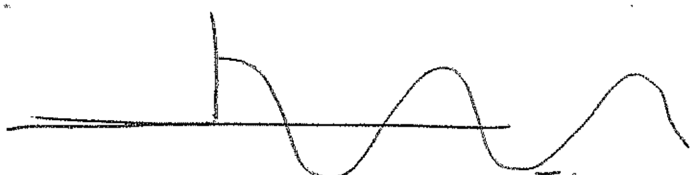
d) $x(t) = \cos(4t) + 3e^{-j12t}$

$T_{0,1} = \pi/2$ $\quad T_{0,2} = \pi/6$ $\quad \frac{T_{0,1}}{T_{0,2}} = \frac{3}{1}$ ✓ $\rightarrow$ periodic

$T_0 = k_0 T_{0,1} = \pi/2$. $\quad f_0 = \frac{2}{\pi}$

e) $T_0 = 4$ , $f_0 = \frac{1}{4}$

(2) a) $x(t) = \cos(t)\,u(t) \rightarrow$ power signal



$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{0}^{T/2} \cos^2(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{0}^{T/2} \frac{1+\cos(2t)}{2}\,dt$$

$$\lim_{T\to\infty} \frac{1}{T} \quad \frac{1}{2}t + \frac{\sin(2t)}{4}\Big|_0^{T/2}$$

$$= \lim_{T\to\infty} \frac{1}{T}\left[\frac{T}{4} + \frac{\sin(T)}{4}\right] = 1/4\,W$$

b) $x(t) = 10\cos(5t)\cos(10t)$

$= 5(\cos(5t) + \cos(15t)) \quad \rightarrow$ periodic $\rightarrow$ power

$P = \frac{25}{2} + \frac{25}{2} = 25\,W \quad$ (power of each sinusoid is $\frac{A^2}{2}$)

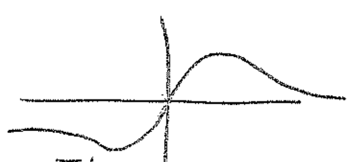
c) $x(t) = \begin{cases} 5\cos(\pi t) & -0.5 \le t \le 0.5 \\ 0 & \text{otherwise} \end{cases}$

$\rightarrow$ finite duration $\rightarrow$ energy $\quad \int_{-0.5}^{0.5} 25\cos^2(\pi t)\,dt$

$$= \frac{25}{2}\left[\int_{-0.5}^{0.5}(1 + \cos(2\pi t))\,dt\right] = \frac{25}{2}\left[t + \frac{\sin(2\pi t)}{2\pi}\Big|_{-0.5}^{0.5}\right]$$

$$= \frac{25}{2}\left[1\right] = 25/2$$

d) $x(t) = t\,e^{-|t|}$



$$E = \lim \int^{T/2} t^2 e^{-2|t|}\,dt = \lim\, 2\int^{T/2} t^2 e^{-2t}\,dt$$

Integration by parts:

$$\underbrace{t^2}_{u}\underbrace{e^{-2t}dt}_{dv} \qquad uv - v\,du$$

$$\frac{t^2e^{-2t}}{-2} + \int \frac{e^{-2t}}{\cancel{2}} \cdot \cancel{2}t\,dt$$

$$\lim_{T\to\infty} -t^2e^{-2t} + 2\underbrace{\int e^{-2t}t\,dt}\Big|_0^{T/2}$$

$$= \lim_{T\to\infty} -t^2e^{-2t} + 2\left[\frac{te^{-2t}}{-2} + \int \frac{e^{-2t}}{2}\,dt\right]\Big|_0^{T/2}$$

$$= \lim_{T\to\infty} -t^2e^{-2t} - te^{-2t} - \frac{e^{-2t}}{2}\Big|_0^{T/2}$$

$$= \lim_{T\to\infty} -\frac{T^2}{4}e^{-T} - \frac{T}{2}e^{-T} - \frac{e^{-T}}{2} + \frac{1}{2} = 1/2$$

$\Rightarrow$ energy signal

e) infinite energy.

$$P = \lim_{T\to\infty} \frac{1}{T}\int_0^{T/2} x^2(t)\,dt$$

let $T = 2 \rightarrow P = 1/2$

$T = 5 \rightarrow P = 2/5$

$T = 9 \rightarrow P = 1/3$

$T = 14 \rightarrow P = 4/14$

$\vdots$

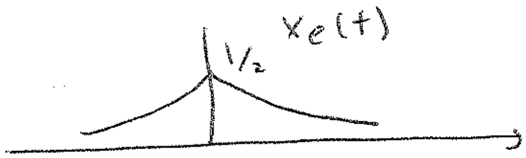
Neither.

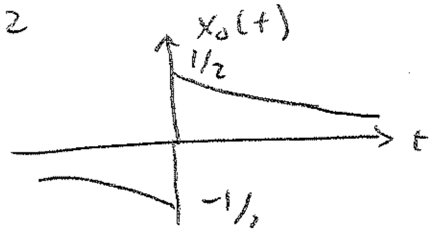
$T$ ~~$n$~~ as $T \to \infty$ $\quad P \to 0 \quad \Rightarrow$ not a power signal.

(3) a) $e^{-2t}u(t)$ Neither

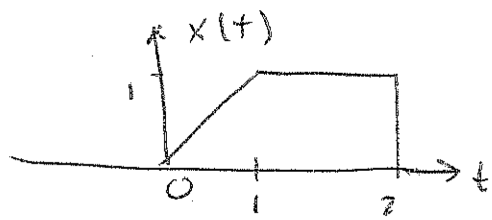
$$x_e(t) = \frac{e^{-2t}u(t) + e^{2t}u(-t)}{2}$$

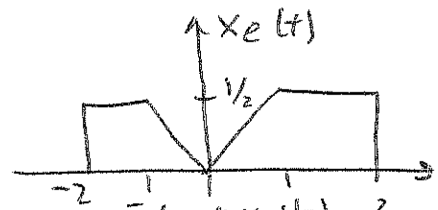
$$x_o(t) = \frac{e^{-2t}u(t) - e^{2t}u(-t)}{2}$$

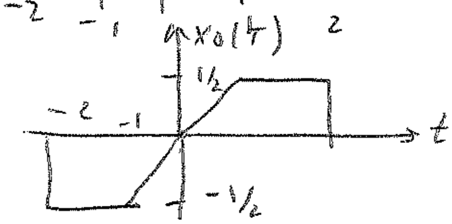



b) Neither



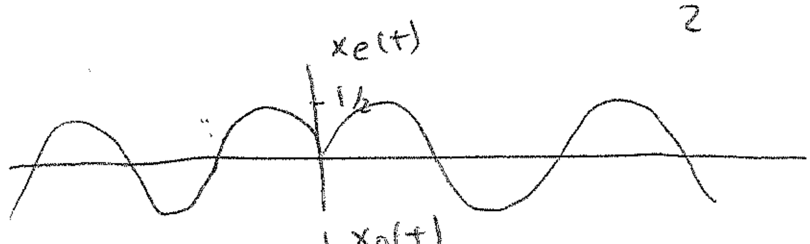



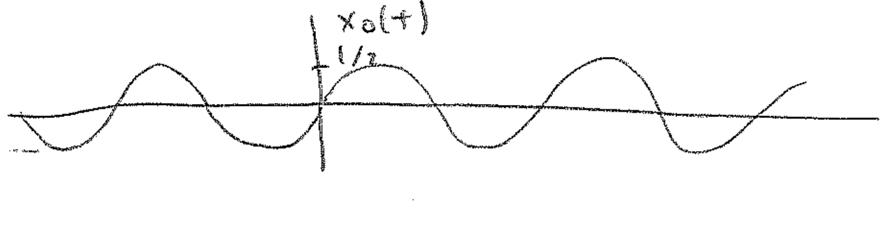




c) $\sin(\pi t)\,u(t)$ Neither

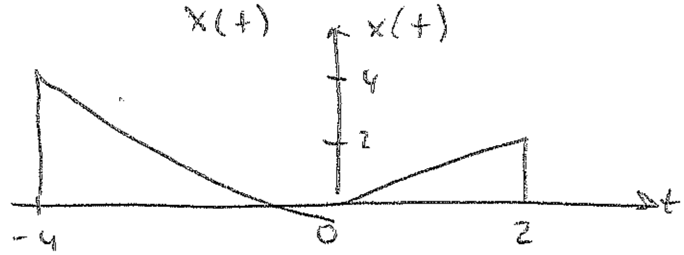
$$x_e(t) = \frac{\sin(\pi t)u(t) + \sin(-\pi t)u(-t)}{2}$$

$$x_o(t) = \frac{\sin(\pi t)u(t) - \sin(-\pi t)u(-t)}{2}$$

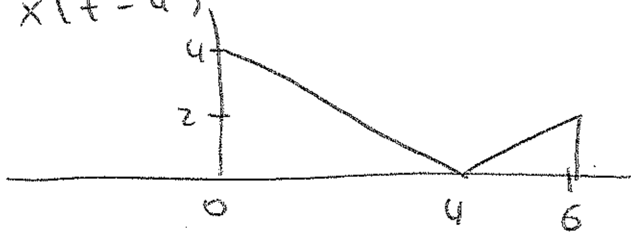


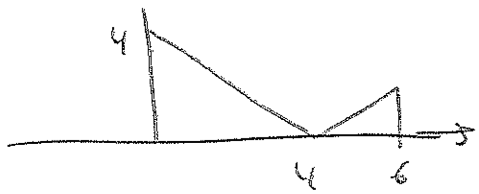
4) $x(t)$



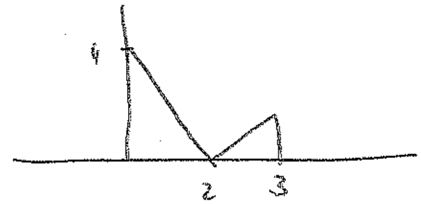

a) $x(t-4)$




b) $x(2t-4) \longrightarrow$ $x(t-4)$



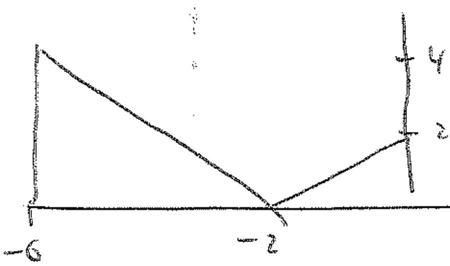

$x(2t-4)$



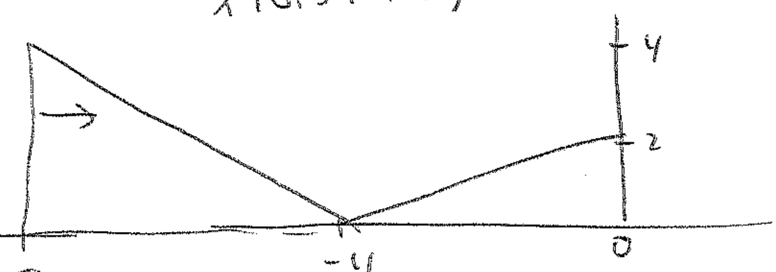

c) $\frac{1}{2}x(0.5t+2)-1$

$x(0.5t+2)$

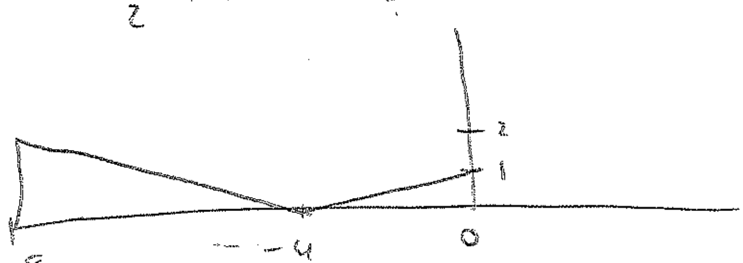
$x(t+2)$



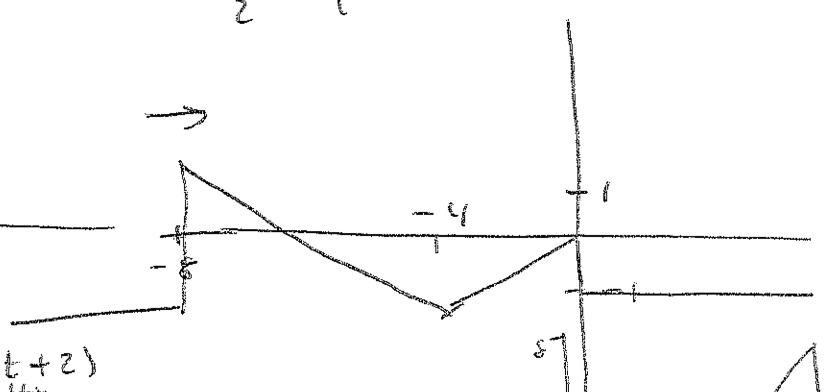

$x(0.5t+2)$




$\frac{1}{2}x(0.5t+2)$

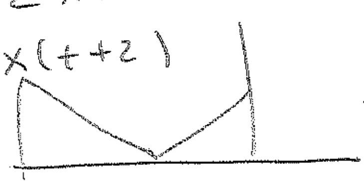



$\frac{1}{2}x(0.5t+2) - 1$

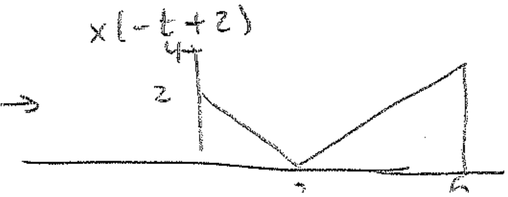


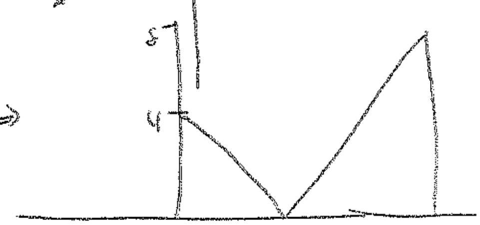

d) $2x(2-t)$

$x(t+2)$



$x(-t+2)$

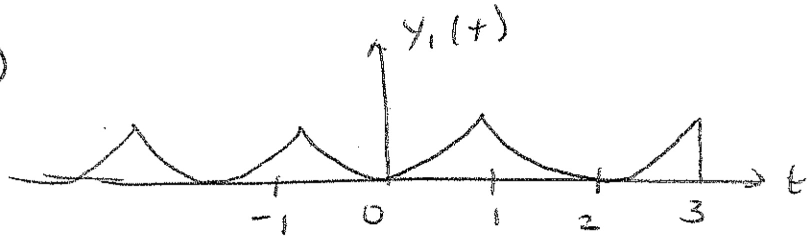


5) 1.3-6

$y_1(t) = y_2(t) = t^2 \quad 0 \le t \le 1$

a)



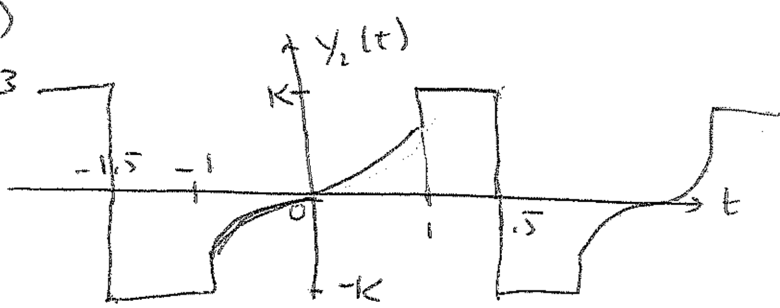

$T_1 = 2$

$$P = \frac{1}{2}\int_{-1}^{1} t^4\,dt = \frac{1}{2}\,\frac{t^5}{5}\Big|_{-1}^{1}$$

$$= \frac{1}{2}\left[\frac{1}{5} + \frac{1}{5}\right] = 1/5 \text{ W.}$$

b) $y_2(t)$

$T_2 = 3$




$$P = \frac{1}{3}\int_{-1.5}^{1.5} y_2^{\,2}(t)\,dt = \frac{1}{3}\left[\int_{-1.5}^{-1} K^2\,dt + \int_{-1}^{1} t^4\,dt + \int_{1}^{1.5} K^2\,dt\right]$$

$$= \frac{1}{3}\left[\frac{2}{5} + 2K^2(0.5)\right] = 1$$

$$= \frac{2}{5} + K^2 = 3 \rightarrow K^2 = \frac{13}{5} \rightarrow K = \frac{\sqrt{13}}{\sqrt{5}}$$

$$y_2(t) = \begin{cases} -\frac{\sqrt{13}}{\sqrt{5}} & -1.5 \le t \le -1 \\ t^2 & -1 \le t \le 1 \\ \frac{\sqrt{13}}{\sqrt{5}} & 1 \le t \le 1.5 \end{cases}$$

c) $y_3(t) = y_1(t) + j\,y_2(t)$

$y_1(t)$ is periodic with $T_1 = 2$ $\rightarrow$ $y_3(t)$ is periodic with $T_3 = 6$

$y_2(t)$ is periodic with $T_2 = 3$

d) $$P = \frac{1}{T_3} \int_{T_3} (y_3(t) y_3^*(t) \, dt$$

$$= \frac{1}{T_3} \int_{T_3} y_1^2(t) + y_2^2(t) \, dt$$

$$= P_{y_1} + P_{y_2} = 1 + 1/5 = 6/5 \, W.$$

6

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t=-0.002:1e-06:0.002;
a=500;
x1=20*sin(2000*pi*t-pi/3).*exp(-a*t);
a=750;
x2=20*sin(2000*pi*t-pi/3).*exp(-a*t);
a=1000;
x3=20*sin(2000*pi*t-pi/3).*exp(-a*t);
subplot(311)
plot(t,x1)
xlabel('Time');
title('a=500');
subplot(312)
plot(t,x2)
xlabel('Time');
title('a=750');
subplot(313);
plot(t,x3);
```