ECE 366, **Exam 1** **Feb. 16, 2015**

Name: ANSWER KEY

Instructions:

- You only need a pen, or a pencil and eraser.
- In particular, this means no calculator or any electronic devices, nor notes, textbooks, etc.
- You must show appropriate legible work and justify your answer to receive full credit.
- There are ~~50~~ 38 possible points. Point values for each problem are as indicated.
- Check and make sure there are four total pages including the cover page, when you begin the exam.
- You must read and sign the honor code below before your test will be graded.

Good Luck!

Problem	Score	Out of
1		7
2		6
3		6
4		11
5		8
Total		38

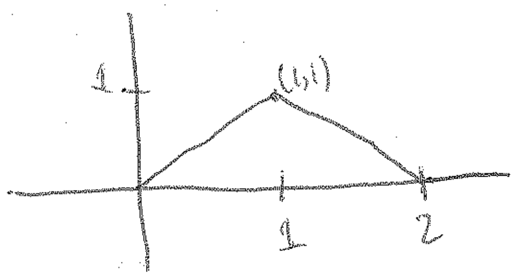
ACADEMIC HONOR CODE

As a student and citizen of the Michigan State University Community I pledge to not lie, cheat, or steal in my academic endeavors.

SIGNED: ANSWER KEY

1. **Basic Signals:** Let $x(t) = r(t) - 2r(t-1) + r(t-2)$, where $r(t) = t\ u(t)$ is the ramp function. [7 *points*]

   (a) Plot $x(t)$. Label all important values on your plot axes. [3 *points*]

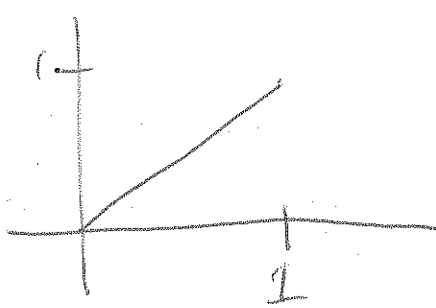


   (b) Plot both the even and the odd parts of $z(t) = t\,[u(t) - u(t-1)]$. Label all important values on your plot axes. [4 *points*]

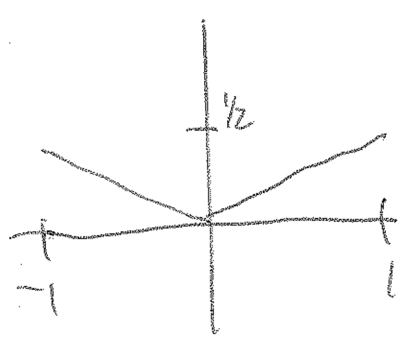
$z_{even} = \dfrac{z(t) + z(-t)}{2}$

$z_{odd} = \dfrac{z(t) - z(-t)}{2}$

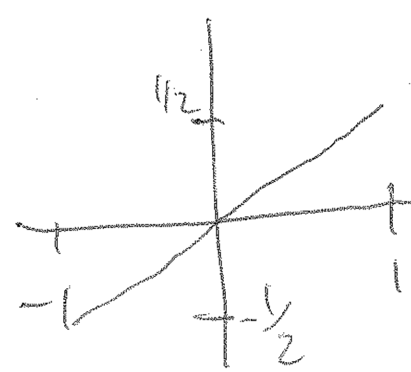
$z(t)$



$\underline{z_{even}}$:



$\underline{z_{odd}}$:



2. **Signal Properties:** Answer both questions below. [6 *points*]

(a) Is $x(t) = u(t+4)\exp(-t)$ a power signal? Show your work. [3 *points*]

$$\lim_{T\to\infty} \frac{1}{T}\int_{-T}^{T}\left[u(t+4)e^{-t}\right]^2 dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T}^{T} e^{-2t}\,u(t+4)\,dt$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{-4}^{T} e^{-2t}\,dt = \lim_{T\to\infty} -\frac{1}{2T}e^{-2t}\Big|_{-4}^{T}$$

$$= \lim_{T\to\infty} -\frac{e^{-2T}}{2T} + \frac{e^{8}}{2T} = 0+0 \rightarrow$$

Not a power signal

(b) Is $x(t) = 5\cos(2t) + \cos\left(\frac{3}{4}t\right) + \sin^2(5t)$ periodic? Show your work. [3 *points*]

$$T_1 = \frac{2\pi}{2}\ ;\ T_2 = \frac{2\pi}{3/4}\ ;\ T_3 = \frac{2\pi}{5}$$

$$\frac{T_1}{T_2} = \frac{4}{3}\cdot\frac{1}{2} = \frac{2}{3},\quad \frac{T_1}{T_3} = \frac{5}{2}$$

rationals! $\Rightarrow$ this signal is periodic

3. **Linearity and Time-Invariance:** For the system described by the following input-output relationship,

$$\frac{d}{dt}\left[y(t)\right] = \int_{-\infty}^{t+2} 5x(\tau)\; d\tau$$

determine the following. [6 *points*]

(a) Is the system linear? Explain your answer, and show all work. [3 *points*]

If $\frac{d}{dt}\left[y_1(t)\right] = \int_{-\infty}^{t+2} 5\,x_1(\tau)\, d\tau$ &

$\frac{d}{dt}\left[y_2(t)\right] = \int_{-\infty}^{t+2} 5\,x_2(\tau)\, d\tau$

Then,

$$\frac{d}{dt}\left[y_1(t)+y_2(t)\right] = \int_{-\infty}^{t+2} 5\left(x_1(\tau)+x_2(\tau)\right) d\tau$$

∴ the system is linear!

(b) Is the system time-invariant? Explain your answer, and show all work. [3 *points*]

$$\int_{-\infty}^{t+2} 5\,x(\tau+t_0)\, d\tau = \int_{-\infty}^{t+2+t_0} 5\,x(\bar{\tau})\, d\bar{\tau}$$

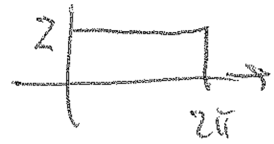
Let $\bar{\tau} = \tau + t_0$

$d\bar{\tau} = d\tau$

$$\frac{d}{dt}\left[y_1(t+t_0)\right] = \int_{-\infty}^{(t+t_0)+2} 5\,x(\bar{\tau})\, d\bar{\tau} = \int_{-\infty}^{t+2} 5x(\tau+t_0)$$

→ it is time invariant!

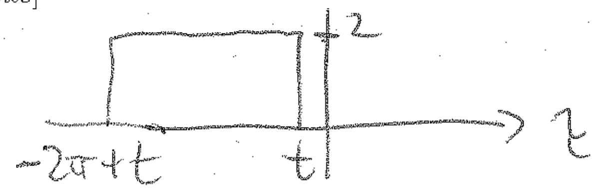
h(t)



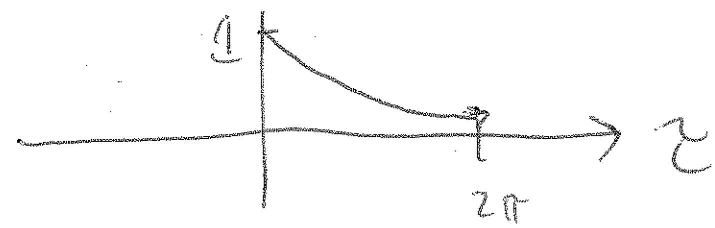
4. **LTI System Responses:** A Linear Time-Invariant System T has the impulse response $h(t) = 2\,[u(t) - u(t-2\pi)]$, where $u(t)$ is the step function. Find the system response $y(t)$ for the input $x(t) = \exp(-t)\,[u(t) - u(t-2\pi)]$ by completing parts $(a) - (d)$ below. [11 *points*]

   (a) Plot both $h(t-\tau)$ and $x(\tau)$ as functions of τ for a fixed t. Label all important values on your plot axes. [6 *points*]

$h(t-\tau)$:



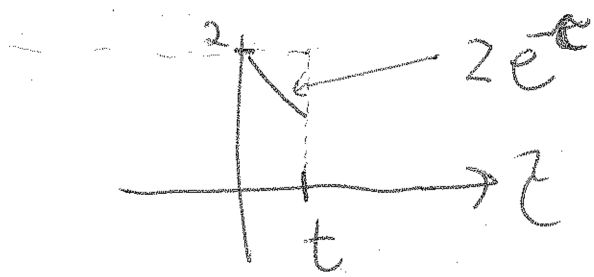
$x(\tau)$:



   (b) Find the system response of T to $x(t)$ for times $t \leq 0$ and $t \geq 4\pi$. Justify your answer. [1 *point*]

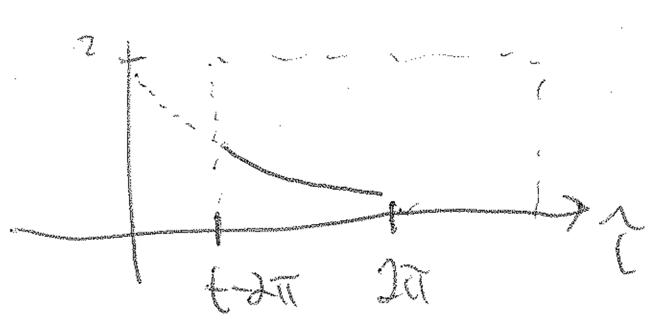
$\boxed{0}$ when $t \leq 0$ or $t \geq 4\pi$ then the nonzero parts of $x(\tau)$ & $h(t-\tau)$ don't overlap.

   (c) Find the system response of T to $x(t)$ for times $0 \leq t \leq 2\pi$. Show your work. [2 *points*]



$$\int_0^t 2e^{-\tau}\,d\tau = -2e^{-\tau}\Big|_0^t = -2e^{-t} + 2.$$

   (d) Find the system response of T to x for times $2\pi \leq t \leq 4\pi$. Show your work. [2 *points*]



$$\int_{t-2\pi}^{2\pi} 2e^{-\tau}\,d\tau = -2e^{-\tau}\Big|_{t-2\pi}^{2\pi} = \left[e^{-(t-2\pi)} - e^{-2\pi}\right]2.$$

5. **Other System Properties:** Consider the Linear Time-Invariant (LTI) system with the following input/output relationship,

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 2)\, d\tau.$$

Answer the following questions about this system. Justify your answers. [8 *points*]

(a) Find the impulse response for this system, $h(t)$. Show your work. [3 *points*]

$$h(t) = \int_{-\infty}^{t} e^{-(t-\tau)} \delta(\tau-2)\, d\tau$$

$$= e^{-(t-2)} \int_{-\infty}^{t} \delta(\tau-2)\, d\tau = e^{-(t-2)} \int_{-\infty}^{t-2} \delta(\tilde{\tau})\, d\tilde{\tau}$$

$\tilde{\tau} = \tau - 2$

$d\tilde{\tau} = d\tau$

$$= \boxed{e^{-(t-2)} u(t-2)}$$

(b) Is this system BIBO stable? Justify your answer. [3 *points*]

Suppose $|x(\tau-2)| \leq M \ \forall \ \tau$. Then..

$$|y(t)| \leq \int_{-\infty}^{t} e^{-(t-\tau)} \cdot |x(\tau-2)|\, d\tau$$

$\tilde{\tau} = \tau - t$

$$\leq M \int_{-\infty}^{t} e^{\cancel{-(t-\tau)}}\, d\tau = M \int_{-\infty}^{0} e^{\tilde{\tau}}\, d\tilde{\tau} = Me^{\tilde{\tau}}\Big|_{-\infty}^{0}$$

$$= M \cdot (1) = \underline{M}$$

Yes— stable!

(c) Is this system causal? Why, or why not? [1 *point*]

Yes— $x(t)$ is only evaluated at times $< t$

(d) Is this system instantaneous/memoryless? Why, or why not? [1 *point*]

No — $x(t)$ is evaluated over all previous times up to $t-2$