CMSE 823, **Computational Practice Problems** **Dec. 6, 2016**

Name: ______________________________

1. **BY HAND - Basic Eigenvalue Estimates:** Let $A \in \mathbb{R}^{3\times 3}$ be

$$A = \begin{pmatrix} 5 & 2 & -1 \\ 2 & 5 & -1 \\ -1 & -1 & 8 \end{pmatrix}.$$

Use Gerschgorin's disk theorem to show that A is positive definite.

2. **BY MATLAB - Householder Reflectors:** Recall that

$$A = \begin{pmatrix} 5 & 2 & -1 \\ 2 & 5 & -1 \\ -1 & -1 & 8 \end{pmatrix}.$$

Produce a tridiagonal matrix $T \in \mathbb{R}^{3\times 3}$ with the same eigenvalues as A.

3. **BY HAND - Bisection Method:** Use the bisection method to count the number of eigenvalues of

$$T = \begin{pmatrix} 5 & 2 & 0 \\ 2 & 5 & -1 \\ 0 & -1 & 8 \end{pmatrix}.$$

in the interval $[6, 9]$.

4. **BY HAND, THEN MATLAB - QR Decomposition:** Find the QR decomposition of the matrix

$$B = \begin{pmatrix} 3 & 4 & 0 \\ 4 & -3 & \sqrt{11} \\ 0 & \sqrt{11} & 0 \end{pmatrix}.$$

Check your answer with MATLAB after you give up.

5. **BY MATLAB - The QR Algorithm:** Use your answer from the last problem to complete one iteration of the QR algorithm on

$$B = \begin{pmatrix} 3 & 4 & 0 \\ 4 & -3 & \sqrt{11} \\ 0 & \sqrt{11} & 0 \end{pmatrix}$$

in order to find $B^{(1)}$. After this single iteration you should already be able to estimate one of B's eigenvalues to within an error of about 0.6 – do so! Produce $B^{(}2)$ and $B^{(}3)$ next, and see if you're eigenvalue estimates tighten up...

6. Find the eigenvalues of

$$A = \begin{pmatrix} 5 & 2 & -1 \\ 2 & 5 & -1 \\ -1 & -1 & 8 \end{pmatrix}$$

any way you like. Then, use the shifted inverse power method (by hand!!) to find the eigenvector for the second largest eigenvalue.