1. Let $A \in \mathbb{C}^{m\times n}$. Use what you know about the relationship between the standard vector inner product and the adjoint of a matrix to show that all the eigenvalues of both A^*A and AA^* are positive real numbers. No use of the SVD allowed!

2. Let $A \in \mathbb{C}^{m\times n}$. Use what you know about the relationship between the standard vector inner product and the adjoint of a matrix to show that all every eigenvalue A^*A is also an eigenvalue of AA^* (and vice versa). No use of the SVD allowed!

3. Do 4.1 on page 30 of Trefethen and Bau.

4. Do 4.4 on page 31 of Trefethen and Bau.

5. Do 5.1 on page 37 of Trefethen and Bau.

6. Do 5.3 on page 37 of Trefethen and Bau.

7. Do 5.4 on page 37 of Trefethen and Bau.

8. Use the singular value decomposition of $A \in \mathbb{C}^{n\times N}$ to help you find a formula for a matrix $A^\dagger \in \mathbb{C}^{N\times n}$ which inverts A on its range. More specifically, write a formula for a matrix $A^\dagger$ that satisfies both $AA^\dagger A = A$ and $A^\dagger AA^\dagger = A^\dagger$. When will $A^\dagger = A^{-1}$ hold?

9. Let $\alpha, \beta \in \mathbb{Z} \setminus \{0\}$. The $\frac{\alpha}{\beta}$-power of a full rank matrix $A \in \mathbb{C}^{N\times N}$ is a matrix $B \in \mathbb{C}^{N\times N}$ with the property that $B^\beta = A^\alpha$. Prove that there always exists a unitary matrix $W \in \mathbb{C}^{N\times N}$ such that the $\frac{\alpha}{\beta}$-power of AW exists. When can W simply be the identity? How can one compute such a B and W for any given $A \in \mathbb{C}^{N\times N}$?

10. **MATLAB INTRO:** Go to the Klein Project Blog and read the article on Google's PageRank algorithm:

    `http://blog.kleinproject.org/?p=280`

    After reading it, go through either the commands in the *PageRank.m* Matlab file (in Matlab), or else the *PageRank.ipynb* Matlab Jupyter notebook (on the JupyterHub), and do the exercise at the very end of the file/notebook for the new matrix R below. Note: R is defined in terms of the matrix P from *PageRank.m* and *PageRank.ipynb*.

    $$R := \begin{pmatrix} \frac{1}{3}P & \frac{1}{4}P & \frac{1}{2}P \\ \frac{1}{3}P & \frac{1}{2}P & \frac{2}{5}P \\ \frac{1}{3}P & \frac{1}{4}P & \frac{1}{10}P \end{pmatrix}. \tag{1}$$

    If you have not used Matlab (much) before, then you should use the Jupyter notebook on the JupyterHub! Turn in the stable distribution you find for this new matrix (or print out a copy of it and turn it in). Which column of R corresponds to the webpage that Google should list first?