1. Let $A \in \mathbb{C}^{m \times n}$. Prove the following *using only the definitions of matrix-vector multiplication, vector addition, and scalar multiplication.*
   (a) Let $\alpha, \beta \in \mathbb{C}$, and $\mathbf{x}, \mathbf{y} \in \mathbb{C}^n$. Prove that $A(\alpha\mathbf{x} + \beta\mathbf{y}) = \alpha A\mathbf{x} + \beta A\mathbf{y}$.
   (b) Let $\mathbf{e}_j \in \mathbb{C}^n$ be such that
   $$(\mathbf{e}_j)_k = \begin{cases} 1 & \text{if } k = j \\ 0 & \text{else} \end{cases}$$
   for all $1 \leq j, k \leq n$. Prove that $A\mathbf{e}_j = \mathbf{a}_j =$ the j^{th} column of A for all $1 \leq j \leq n$.
   (c) Prove that
   $$A\mathbf{x} = x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \cdots + x_n\mathbf{a}_n = \sum_{j=1}^{n} x_j\mathbf{a}_j$$
   for all $\mathbf{x} \in \mathbb{C}^n$. You can use (a) and (b) above to save time.
   (d) Using (c) above, prove that the range of A as a function from $\mathbb{C}^n$ to $\mathbb{C}^m$ is exactly $C(A)$.
2. Do 1.1 on page 9 of Trefethen and Bau.
3. Do 1.4 on page 10 of Trefethen and Bau.
4. Do 2.1 on page 15 of Trefethen and Bau.
5. Do 2.2 on page 15 of Trefethen and Bau.
6. Do 2.3 on page 15 of Trefethen and Bau.
7. Do 2.4 on page 16 of Trefethen and Bau.
8. Do 2.6 on page 16 of Trefethen and Bau.
9. Do 3.3 on page 24 of Trefethen and Bau.
10. Do 3.6 on page 24 of Trefethen and Bau.