NAME:_____

Math 481 **Quiz 28** **Apr 1, 2026**

PROPOSITION: Let G be a connected graph with a cycle C having edge $e = xy$, so $e \subset C \subset G$. Then the graph $G - e$, removing e but keeping vertices x, y, is connected.

Idea: Need that any vertices u, v are connected in $G-e$, along a path which does not use $e = xy$. Since G is connected, there is a path uPv which might use $e = xy$ in C. In that case, avoid e by going the other way around C, getting a new path $uP'v$ which connects u to v in $G - e$.

PROOF with FRAMING RUBRICS.

HYPOTHESIS

Assume $G = (V, E)$ is connected and contains a cycle C, which contains an edge $e = xy$.

UNPACK HYPOTHESIS

We denote the cycle $C = xeyQx$, from x along e to y, then along a path Q to x.

Since G is connected graph, any vertices u, v are connected by a path $uPv \subset G$.

Let $G-e = (V, E - \{e\})$ be the graph with the same vertices as G but missing edge e.

DEDUCTIVE BRIDGE

If the path uPv does not contain e, then $P' = P$ is a path $vP'w \subset G-e$.

If uPv does contain $e = xy$, we may write it as vP_1xeyP_2w, and construct

$$uWv = vP_1xQyP_2w,$$

the walk from v along P to x, then around Q to y, continuing along P to w. By definition, the path P and the cycle C can use vertices x, y only once, so the path segments vPx, xQy, yPw cannot contain e. Thus W is a walk in $G-e$ connecting v, w.

But we know that we can cut any walk down to a path by removing all steps between repeated vertices. Thus we can cut down the walk uWv to a path $uP'v \subset G-e$.

UNPACK CONCLUSION

Either way, arbitrary vertices v, w are connected by $vP'w$ in $G-e$.

CONCLUSION

$G - e$ is connected.

Note: The framing rubrics like HYPOTHESIS are just to guide you in constructing the proof. Leave them out of your finished homework.