Here are some subtle points that many people missed.

2.9 *Proposition:* $\left\{ {k \atop n} \right\} = \left\{ {k-1 \atop n-1} \right\} + n \left\{ {k-1 \atop n} \right\}$.

Letting $\mathcal{S}_k^{(n)}$ be the class of set partitions $\{S_1, \ldots, S_n\}$ with $S_1 \sqcup \cdots \sqcup S_n = [k]$, we define a bijective transformation:

$$T : \mathcal{S}_k^{(n)} \xrightarrow{\sim} \mathcal{S}_{k-1}^{(n-1)} \ \coprod \ [n] \times \mathcal{S}_{k-1}^{(n)}.$$

If $S_i = \{k\}$, we omit the i^{th} basket:

$$T\{S_1, \ldots, S_n\} = \{S_1, \ldots, S_{i-1}, S_{i+1}, \ldots, S_n\}.$$

Otherwise, suppose $k \in S_i$ but $|S_i| > 1$. Since the S_i's are not ordered, we may place them in a standard order, for example so that $\min(S_1) < \cdots < \min(S_n)$. We then let:

$$T\{S_1, \ldots, S_n\} = (i) \times \{S_1, \ldots, S_i \backslash \{i\}, \ldots, S_n\},$$

removing ball k from the i^{th} basket, and recording the position from which it was removed. (Note that removing k will not change the standard order on the resulting sets.) We can easily define the inverse, which proves the bijectivity, and hence the recurrence formula. $\square$

The point is that we must specify how to record the position of the removed element, even though the baskets are unordered.

2.7 For $\left\{ {k \atop \leq n} \right\} = \left\{ {k \atop 1} \right\} + \cdots + \left\{ {k \atop n} \right\}$, arrangements of k labeled balls in n interchangeable baskets, we have the following recurrence. Suppose the k^{th} ball is in basket S, which contains i balls. This arrangement is equivalent to arranging the remaining $k-i$ balls in $n-1$ baskets, and also choosing $S \backslash \{k\} \subset [k-1]$. Thus:

$$\left\{ {k \atop \leq n} \right\} = \sum_{i=1}^{k} \binom{k-1}{i-1} \left\{ {k-i \atop \leq n-1} \right\}.$$

This is a decent recurrence for computing $\left\{ {k \atop \leq n} \right\}$ in terms of smaller numbers of the same kind, although the right side uses binomial coefficients as well.

3. In each case, the given transformation T is injective, but not surjective: that is, we can define an inverse transformation T^{-1} on the image of T, but it not on the whole codomain.

a. The transformation $T(\lambda_1 \geq \ldots \geq \lambda_k) = \{\lambda_k+1, \ldots, \lambda_1+1\}$ only outputs multisets $M = \{s_1 \leq \cdots \leq s_k\}$ with $s_1 + \cdots + s_k = \lambda_1 + \cdots + \lambda_k + k = n + k$, so there are many other multisets in $\left(\!\binom{n}{k}\!\right)$ which are not hit.

b. The transformation which takes $(\lambda_1 \geq \cdots \geq \lambda_k)$ to a multiset with multiplicities $m_i = \lambda_i$ only hits the multisets with $m_1 \geq m_2 \geq \cdots$. Any other multiset in $\left(\!\binom{k}{n}\!\right)$ is not hit.

c. Here transformation takes $(\lambda_1 \geq \cdots \geq \lambda_k)$ to $(\lambda_1 \geq \cdots \geq \lambda_{k-1})$ if $\lambda_k = 0$, and to $(\lambda_1 \geq \cdots \geq \lambda_{k-1} \geq \lambda_k-1)$ if $\lambda_k > 1$. In fact, we always have $\lambda_{k-1} > \lambda_k-1$, so the transformation misses those partions $(\lambda'_1 \geq \cdots \geq \lambda'_k)$ with $\lambda'_{k-1} = \lambda'_k$.