NAME:____________________________

Math 481 **Quiz 9 Solution** **Feb 2, 2026**

A candy machine sells 4 kinds: Kit Kat, M&M's, Snickers, Twix. (No supply limit.)

How many possibilities for 10 sucessive purchases with *at least one of each kind.*

GOOD: $M, T, T, S, S, K, M, T, S, M$ BAD: $M, T, T, S, S, T, M, T, S, M$ (missing K)

1. Set up the Principle of Inclusion-Exclusion. Define a set of all cases A and sets of bad cases B_1, B_2, B_3, B_4. Use **sets, not numbers**: don't jump to the final numerical answer.

Note: Though not stated, it is reasonable to assume that order matters in the 10 choices, because: (i) the problem says "10 successive purchases"; (ii) examples are given in random order rather than alphabetical order; (iii) probability of purchasing 10 random items with at least one of each is correctly computed from counting ordered choices, not unordered.

$$A = \{ \text{ all sequences of 10 independent choices from 4 kinds } \}$$

$$B_1 = \{ \text{ sequences with no K } \}$$

$$B_2 = \{ \text{ sequences with no M } \}$$

$$B_3 = \{ \text{ sequences with no S } \}$$

$$B_4 = \{ \text{ sequences with no T } \}$$

2. State the PIE equation in this case in terms of $|A|$, $|B_1|$, etc.
Give the general formula using set letters, not the final numerical answer!

$$\left| A \setminus \bigcup_{i=1}^{4} B_i \right| = |A| - \sum_i |B_i| + \sum_{i<j} |B_i \cap B_j| - \sum_{i<j<k} |B_i \cap B_j \cap B_k| + |B_1 \cap B_2 \cap B_3 \cap B_4|.$$

3. Finally, solve the problem by numerically evaluating the terms in the PIE formula. Your final answer should use symbols like $\binom{n}{k}$, with no summations.

$|A| = 4^{10}$ by the Multiplication Principle; and $|B_i| = 3^{10}$ since if one kind is missing, there is a free choice from 3 kinds; etc. The number of terms in each summation comes from choosing an intersection of some number of the 4 sets B_1, B_2, B_3, B_4.

$$\left| A \setminus \bigcup_{i=1}^{4} B_i \right| = 4^{10} - \binom{4}{1} 3^{10} + \binom{4}{2} 2^{10} - \binom{4}{3} 1^{10} + \binom{4}{4} 0^{10}$$

$$= 4^{10} - 4(3^{10}) + 6(2^{10}) - 4.$$