Cayley graph of A_4 = Rotations of Tetrahedron

Generators $R_1 = \begin{pmatrix}1&2&3&4\\1&4&2&3\end{pmatrix} = (243)$ (blue arrow) $\quad R_1^3 = I$

$R_2 = \begin{pmatrix}1&2&3&4\\3&2&4&1\end{pmatrix} = (134)$ (red arrow) $\quad R_2^3 = I$ } triangles

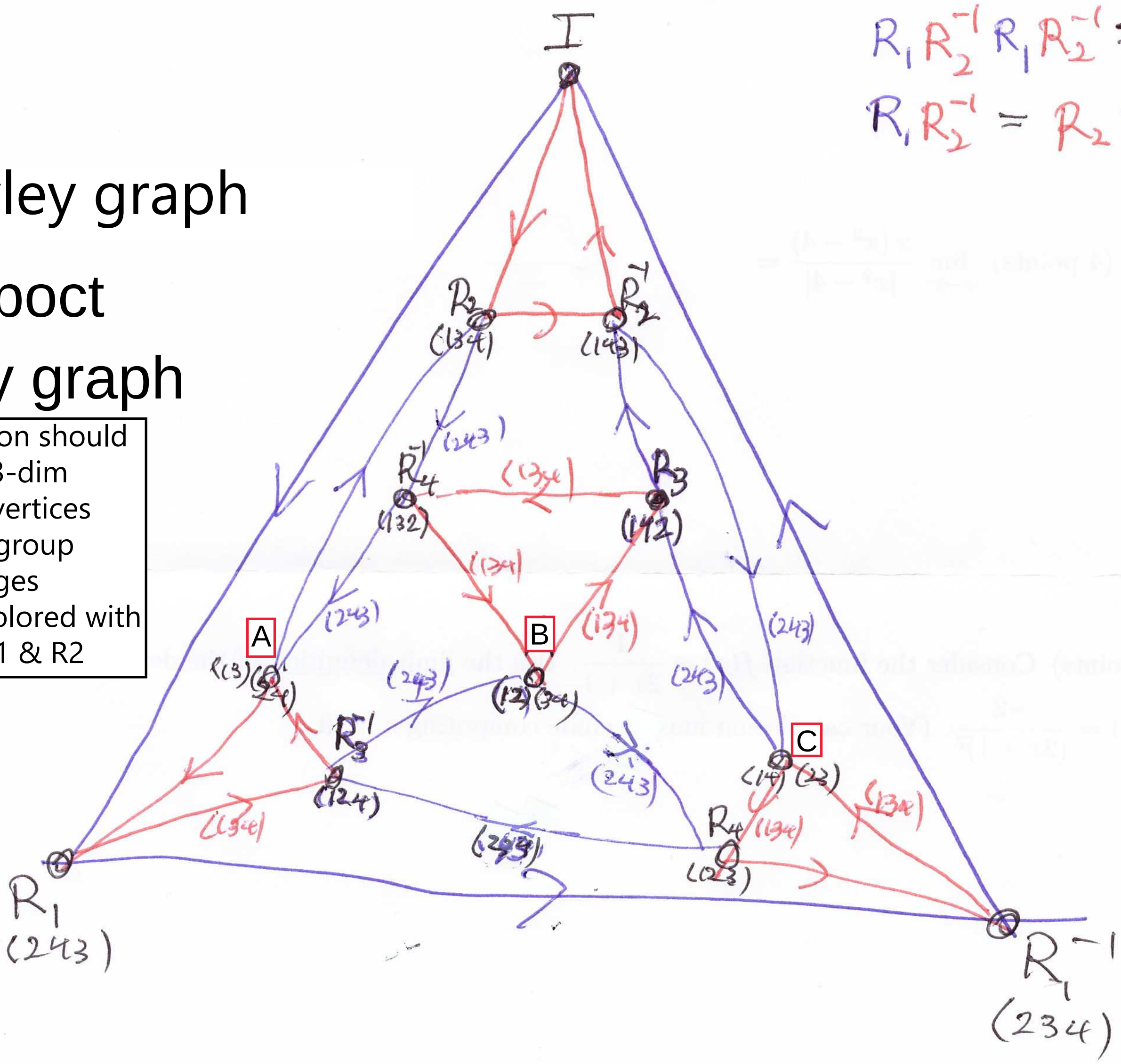
$R_1 R_2^{-1} R_1 R_2^{-1} = I$ } rectangles

$R_1 R_2^{-1} = R_2 R_1^{-1}$

#8 Cayley graph

#9 Cuboct Cayley graph

Cuboctahedron should be drawn in 3-dim perspective; vertices labeled with group elements; edges oriented & colored with generators R1 & R2



Edge-graph of Cuboctahedron

12 vertices

8 triangle faces

6 rectangle faces

#3,4,6

B $= R_1 R_2 R_1 = R_2 R_1 R_2 = (12)(34)$

A $= R_2 R_1^{-1} = R_1 R_2^{-1} = (13)(24)$

C $= R_1^{-1} R_2 = R_2^{-1} R_1 = (14)(23)$

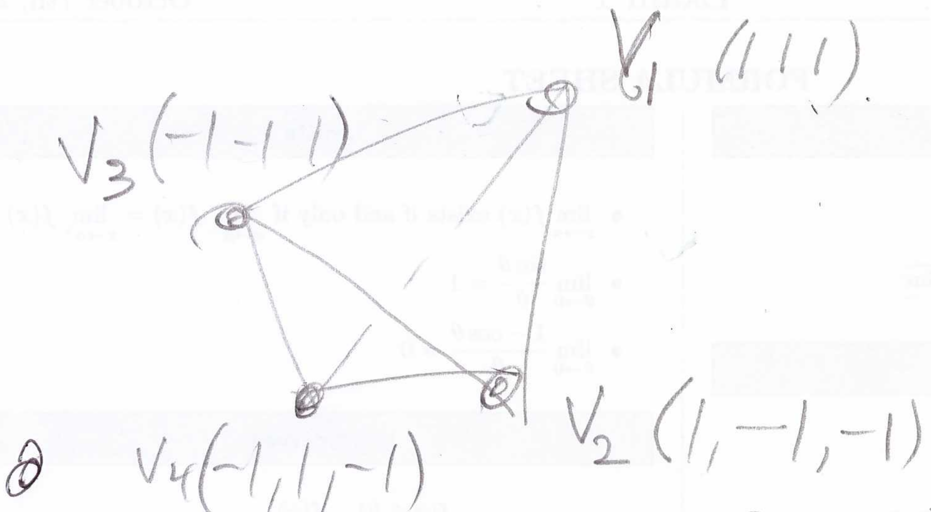
$R_3 = R_2^{-1} R_1^{-1} = R_2 R_1 R_2^{-1} = (142)$

$R_4 = R_1^{-1} R_2^{-1} = R_2^{-1} R_1 R_2 = (123)$

#7 (1pt)

Show |G| = 12. Clearly G subset S_4. For permutation p in G, we have p(1) in {1,2,3,4} and p(2) in {1,2,3,4}-{p(1)}. But p(3) is the unique vertex clockwise from arrow p(1)-->p(2), since rotation maintains orientation. Total choices for 4 vertices: (4)(3)(1)(1) = 12.

#5 Eigenvector



$(143)\ R_2^{-1}$

(234)

$R_2^2 R_1 = (143)(234) = (142) = R_3$ $\xrightarrow{R_1\,(234)}$ $(14)(23) = C$

#1

#2

It's OK to label R1 as R1^(-1) etc, and vary labeling of A,B,C. But labels must be consistent, perms agree with matrices.

$R_1 = (243)$ $(100) \to (010) \to (001)$ $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$R_2 = (134)$ $(100) \to (0\,{-1}\,0) \to (0\,0\,{-1})$ $\begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$R_3 = (142)$ $({-1}\,0\,0) \to (0\,{-1}\,0) \to (001)$ $\begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$

$R_4 = (123)$ $({-1}\,0\,0) \to (010) \to (0\,0\,{-1})$ $\begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$

$R_1 R_2 = (243)(134) = (124) = R_3^{-1}$

$R_2 R_1 = (134)(243) = (132) = R_4^{-1}$

$R_1 R_3 = (243)(142) = (132) = R_1^{-1}$

$R_3 R_1 = (142)(243) = (143) = R_2^{-1}$

$R_1 R_4 = (243)(123) = (143) = R_2^{-1}$

$R_3 R_1 = R_1 R_4$ / $R_1^{-1} R_3 R_1 = R_4$

$R_1 R_2^{-1} = (243)(143) = (13)(24) = $ A $= R_2 R_1^{-1}$