Math 299 **Recitation 5** **Sept. 26, 2013**

PROBLEM 1. Say whether True or False for each of the following and if the following statement is not true, fix the statement to be true.

1. ______ : "Earning a final grade of C from MTH 299 class is a necessary and sufficient condition for passing MTH 299".
2. ______ : "Competing all the requirements of a given degree program is a necessary and sufficient condition for earning the degree".
3. ______ : "x is a square" is a sufficient condition for "x is a rectangle".
4. ______ : "x is an equilateral rectangle" is a necessary condition for "x is a square".
5. ______ : "Being divisible by 4 is necessary for being an even number".
6. ______ : "A function f is continuous at a point a" is necessary for "The limit of the funciton f exists as x approaches to a".
7. ______ : " x is an element in a set $A \cap B^c$ " is sufficient for " x is an element in a set $A \setminus B$ ".
8. ______ : "A function f is invertible" is necessary for "A function f is injective".

PROBLEM 2. Fill in the blank with *necessary, sufficient*, or *necessary and sufficient*.

"a and b are perfect squares " is ______________________ for "ab is a perfect square ". (Note: Perfect square is an integer that is the square of an integer.)

(1) Restate the above statement as "*if* $\cdots$, *then* $\cdots$".

(2) Write the *Hypothesis* and the *Conclusion*.

(3) Prove the statement with mathematical rigor. If the answer is not necessary or not sufficient, find an counterexample.

PROBLEM 3. (Bonus Problem) Fill in the blank with *necessary, sufficient*, or *necessary and sufficient*.

"$x \in (A \cap B)$ " is ________________________ for "$x \in (A^c \cup B)$".

(1) Restate the above statement as "*if* $\cdots$, *then* $\cdots$".

(2) Write the *Hypothesis* and *Conclusion*.

(3) Prove the statement with mathematical rigor. If the answer is not necessary or not sufficient, find a counterexample.

Adjust space above and pick only 5 of the following *before printing*.

PROBLEM 4. Use quantifiers to express the following statements.

A. Every student in MTH299 knows how to find the negation of a statement.

B. Every student experiences moments of joy.

C. I know someone who seems never to have experienced any moments of shyness in their whole life.

D. Every student at MSU is at least 18 years old.

E. If you disagree with a statement D, write negation of the statement D and again express it with quantifiers.

F. For any natural number n, there is a prime number m larger than n.

G. There is an integer M larger than the number of contacts saved in any MSU student's cell phone.

H. There are at least two students at MSU whose lastnames are "Sage".

PROBLEM 5. (A substitute problem in case that lecture doesn't cover up to negation of quantifiers) Quantifiers in sentences are one of the linguistic constructs that are hard for computers to handle in general. Here is a nice pair of example dialogues :

1. A: "How was the birthday party after I left?"

   B: "It was really fun. Everybody had a drink."

2. A: "How was the birthday party after I left?"

   B: "It was really fun. Everybody watched a movie."

Did everybody have their own drink, or did they share the same drink? Did they watch the same movie or a different movie (with their smart phone)? Write the precise situation in the sentence B with quantifiers. The order of quantifiers are important to interpret the situation correctly.

PROBLEM 5. An expression involving quantifieres is in *positive form* if none of the quantifiers is negated. Thus $\neg\forall x,\ P(x)$ is not in positive form, but the equivalent expression $\exists x,\ \neg P(x)$ is in positive form. Negate each of the following statements and express it in positive form.

1. $\forall x \in \mathbb{N}\ \exists y \in \mathbb{N},\ x + y = 1.$

2. $\forall x > 0\ \exists y < 0,\ x + y = 0.$

3. $\exists x \in \mathbb{R}\ \forall \epsilon > 0,\ -\epsilon < x < \epsilon.$

4. $\forall x, y \in \mathbb{N}\ \exists z \in \mathbb{N},\ x + y = z^2.$

PROBLEM 6. Go Spartan! There is a tailgate party at the parking lot of a stadium. A group of seven students, Alex, Bob, Cathy, David, Ellena, Fernando, and George, are going to the tailgate together. They will drink soda subject to the following:

> If Fernando drinks coke, then Alex doesn't drink it.
>
> Ellena doesn't drink coke only if George drinks it.
>
> If Cathy drinks coke, then so does Fernando.
>
> "David doesn't drink coke" is sufficient for "Bob drinks it".
>
> "Cathy drinks coke" is necessary for "David does drink coke".

Which one of the following could be a complete and accurate list of students who drink coke? Remember that the conditional statement (original statement) is logically equivalent to its contrapositive. In addition, use the fact that if A implies B and B implies C, then A implies C.

(1) Alex and Ellena

(2) Cathy and David

(3) Bob and Fernando

(4) Ellena, David, and George

(5) Bob, Ellena, Fernando, and George.