Math 299 **Recitation 4** **Sep 19, 2013**

PROBLEM 1. Use truth tables to prove that the statements "not(A and B)" and "(not A) or (not B)" are equivalent.

A	B	not(A)	not(B)	A and B	not(A and B)	not(A) or not(B)
T	T					
T	F					
F	T					
F	F					

PROBLEM 2. (**Bonus problem** - can be left for homework.) Use Problem 1 to prove that "not(A or B)" and "(not(A) and not(B))" are equivalent **without writing out truth tables**.

Step 1. Note that if A is equvalent to B then not(A) is equivalent to not(B) and vice versa.

Step 2. Using Step 1, show that "C and D" is equvalent to "not(not(C) or not(D))".

Step 3. Now apply Step 2 to the statement C substituted with not(A) and the statement D substituted with not(B).

PROBLEM 3. Use *not*, *and* and *or* to define the following set operations.

(a) A^c (b) $A \cap B$ (c) $A \cup B$ (d) $A \setminus B$ (e) $(A \cap B)^c$ (f) $A^c \cup B^c$ (g) $(A \cup B)^c$

SOLUTION:

(b) The statement "$x \in A \cap B$" is equivalent to the statement "$x \in A$ and $x \in B$."

(g) The statement "$x \in (A \cup B)^c$" is equivalent to the statement "not($x \in A$ or $x \in B$)."

PROBLEM 4. (**Bonus problem** - can be left for homework.) Combine Problems 1-3 to prove De Morgan's laws for sets.

(a) $(A \cap B)^c = A^c \cup B^c$

(b) $(A \cup B)^c = A^c \cap B^c$ (Uses Problem 2.)

PROBLEM 5.Identify the hypothesis and conclusion in each statement, and give the simplified negation of each statement. *Simplified negation* means the "not" has been fully distributed through the other operations and into the atomic statements. For example, the negation of "I will not quit and I will win" is: "I will quit or I will lose".

(a) If $f'(x)$ is positive on (a, b), then $f(b) > f(a)$.

(b) The quadratic equation $ax^2 + bx + c = 0$ (with $a, b, c \in \mathbb{R}$) has two real solutions, assuming its discriminant $b^2 - 4ac$ is positive.

(c) The set $\{x \in \mathbb{R} \mid x^2 + a = 0\}$ is nonempty only if $a \leq 0$.

(d) For a function f to be continuous at a point c, it is necessary that $\lim_{x \to c-} f(x) = \lim_{x \to c+} f(x)$.

(e) For $x = c$ to be a vertical asymptote, it is sufficient that $\lim_{x \to c+} f(x) = -\infty$.

PROBLEM 6. Use truth tables to show that "$A \Rightarrow B$" is equivalent to "$not(B) \Rightarrow not(A)$" and is not equivalent to "$not(A) \Rightarrow not(B)$." Note that the second statement is called the **contrapositive** of the first. We will revisit this later.

A	B	not(A)	not(B)	A ⇒ B	not(B) ⇒ not(A)	not(A) ⇒ not(B)
T	T					
T	F					
F	T					
F	F					