Math 299 **Homework 9/4** **Fall 2013**

1. Let S be the set of students in a class. Suppose that the class wants to select a class president, vice president, and a secretary (with no one holding two positions).
   (a) Model this situation with set & list notation: define a set T whose elements represent all possible choices for the three positions.
   (b) What is the cardinality of T if $|S| = 3$: that is, how many ways to choose the three positions if there are only three students in the class? What is the cardinality of T if $|S| = 4$ (four students in the class)?

2. (a) Let $S = \{a, b, c, d\}$ be a set with four elements. List the subsets of S that have cardinality 2 (that is, all two-element subsets). How many such subsets are there?
   (b) Alice, Bob, Carol, and Dave all exchange handshakes (Alice & Bob, Alice & Carol, …, Carol & Dave). Model this situation formally. That is, consider the set/list data needed to represent one handshake, and define a set representing all the handshakes. How many handshakes occurred? How does this relate to part (a)?
   (c) How does the relation between (a) and (b) generalize to sets with n elements? (Note: Next week we will learn the general formula for the number of possible handshakes amongst n people).

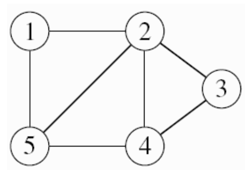
3. (a) For each of the sets S below, list all of the subsets (that is, write out the power-set of S). Do you notice a pattern to the number of subsets? (We will develop techniques to help you prove your guess.)
   i. $S = \emptyset$
   ii. $S = \{a\}$
   iii. $S = \{a, b\}$
   iv. $S = \{a, b, c\}$

   (b) For each of the nonempty sets in (a), list all of the possible functions

$$f : S \to \{0, 1\}.$$

   How many functions are there for each S? Is the relation with (a) a coincidence? Hint: Write functions using the ordered pair definition in Supplement 9/4 p.3. For example, the function $f : \{a, b\} \to \{0, 1\}$ defined by $f(a) = 1$, $f(b) = 0$ is written as: $f = \{(a, 1), (b, 0)\}$.

4. A *combinatorial graph* (not to be confused with the graph of a function) is a mathematical object to model real-world situations in which certain pairs of discrete objects are related or attached to each other.



In this picture, the objects are the vertices or nodes $1, 2, \ldots, 5$; and 1 & 2 are attached, 1 & 5 are attached, etc., but 1 & 3 are *not* attached. We formalize this as a set of vertices $V = \{1, 2, 3, 4, 5\}$, and a set of edges E whose elements are unordered pairs of attached vertices: $e = \{v, w\}$. That is:

$$E = \{\{1, 2\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 4\}, \{4, 5\}\}.$$

In general, we define a graph G as any pair of sets (V, E) in which the elements of E are two-element subsets of V: $e = \{v, w\}$ with $v, w \in V$.

(a) Suppose that we have five people - Alice, Bob, Carol, Dave, Eve. Alice knows everyone (and everyone knows Alice). Bob and Carol know each other. Dave and Eve know each other. Draw a graph that models this situation, and write its formal data (V, E).

(b) Think of substantially different real-world situations naturally modeled by a graph.