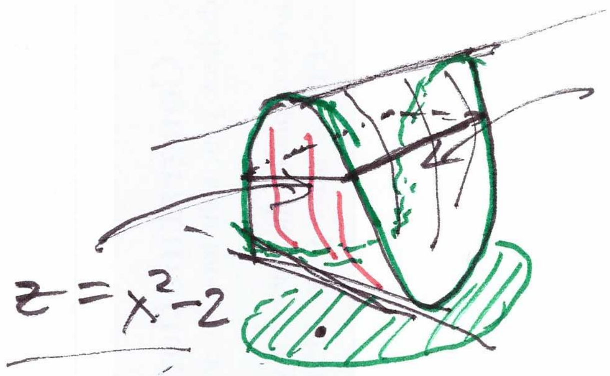
Math 254H 4/24 Solid between $z = 2 - y^2$

$z = x^2 - 2$



Parametrize?

$P(u,v,w) = (x,y,z)$

Use height z as a parameter.

slice: z = fixed, $x^2 - 2 \le z \le 2 - y^2$

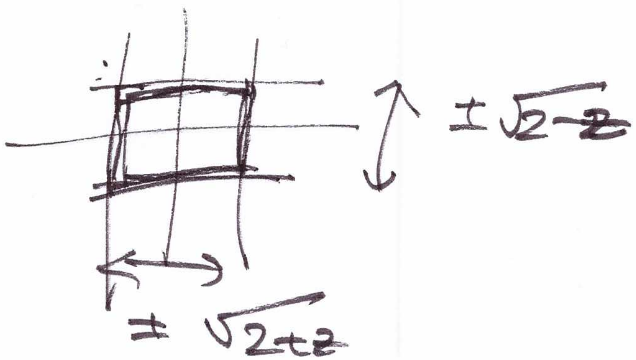
$-2 \le z \le 2$

$z \le 2 - y^2 \implies y^2 \le 2 - z$

$|y| \le \sqrt{2-z}$

$x^2 - 2 \le z \implies x^2 \le 2 + z$

$|x| \le \sqrt{2+z}$



$(u,v,w) \in [-1,1]^3$

$z = 2w$

Simple region $\left\{ \begin{array}{c} -2 \le z \le 2 \\ -\sqrt{2+z} \le x \le \sqrt{2+z} \\ -\sqrt{2-z} \le y \le \sqrt{2-z} \end{array} \right\}$

No $P(u, v, w)$

No stretching!

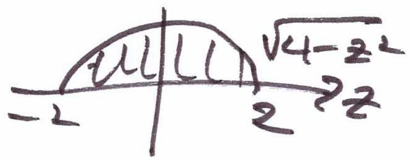
$$\underline{Vol} = \int_{z=-2}^{2} \underbrace{\int_{x=-\sqrt{2+z}}^{\sqrt{2+z}} \cdots \int_{y=-\sqrt{2-z}}^{\sqrt{2-z}}}_{\text{area of slice rectangle}} \cdots 1 \quad \underline{dx\, dy\, dz}$$

$$= \int_{z=-2}^{2} 4\sqrt{(2+z)(2-z)}\, dz = 4\int_{z=-2}^{2} \sqrt{4-z^2}\, dz$$

$z = 2\sin\theta$

$\sqrt{4-z^2} = 2\cos\theta$

$$= 4\cdot 2\int_{-2}^{2} - - - .$$



$$= 4\cdot \frac{1}{2}\pi 2^2 = 8\pi$$

Analyze xy-shadow : shadow of boundary curve

Intersect $z=2-y^2$, $z=x^2-2$ i.e. $2-y^2=x^2-2$

$\boxed{x^2+y^2=4 \quad z=2-y^2}$ circle

Parametrize: $(r,\theta,z) \xrightarrow{Cyl} (x,y,z)$

Cylindrical coords

$R^* \longrightarrow R$

$$Cyl(r,\theta,z)=(r\cos\theta,\ r\sin\theta,\ z)$$

R^*

$$0\le r\le 2$$
$$0\le\theta\le 2\pi$$
$$r^2\cos^2\theta-2 \le z \le 2-r^2\sin^2\theta$$

$$x^2-2 \le z \le 2-y^2$$

$\det[D(Cyl)]$

stretch

$$\text{Vol} = \int_{r=0}^{2}\int_{\theta=0}^{2\pi}\int_{z=r^2\cos^2\theta-2}^{2-r^2\sin^2\theta} r \; dz\, d\theta\, dr$$

no z

$$=\int_{r=0}^{2}\int_{\theta=0}^{2\pi} r(4-r^2\sin^2\theta-r^2\cos^2\theta)$$
$$r(4-r^2)\; d\theta\, dr$$

$$= \int_{r=0}^{2}\int_{\theta=0}^{2\pi} 1\cdot\underbrace{(4r-r^3)}_{\text{no } \theta}\, d\theta\, dr$$

$$= \left(\int_{\theta=0}^{2\pi} \mathbf{1}\, d\theta\ d\theta\right)\left(\int_{r=0}^{2}(4r-r^3)\,dr\right) \overset{?}{=} 8\pi$$

Pillow ~~solid~~ surface: Each horizonal slice is ellipse

thickness

0 . $z=1$

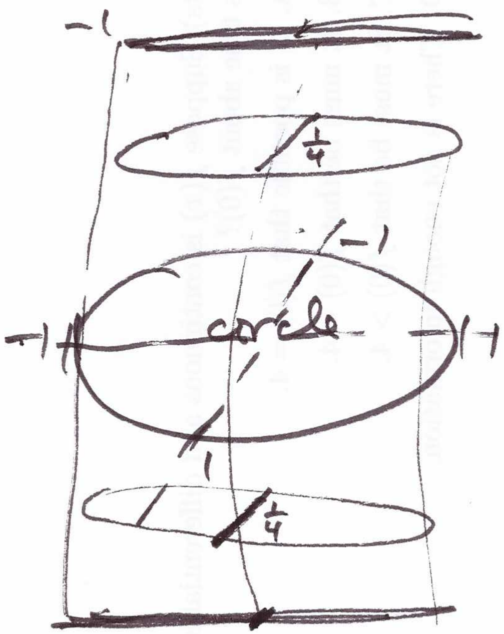
$\frac{1}{4}$.

1 . $z=0$

$\frac{1}{4}$.

0 $z=-1$

$(1-z^2) \leftarrow z$



Parametrize

& draw it!

General picture

of parametrization?

$P(u,v)$ "cylindrical"

football

$P(\theta, z) \stackrel{?}{=} ((1-z^2)\cos\theta, (1-z^2)\sin\theta, z)$

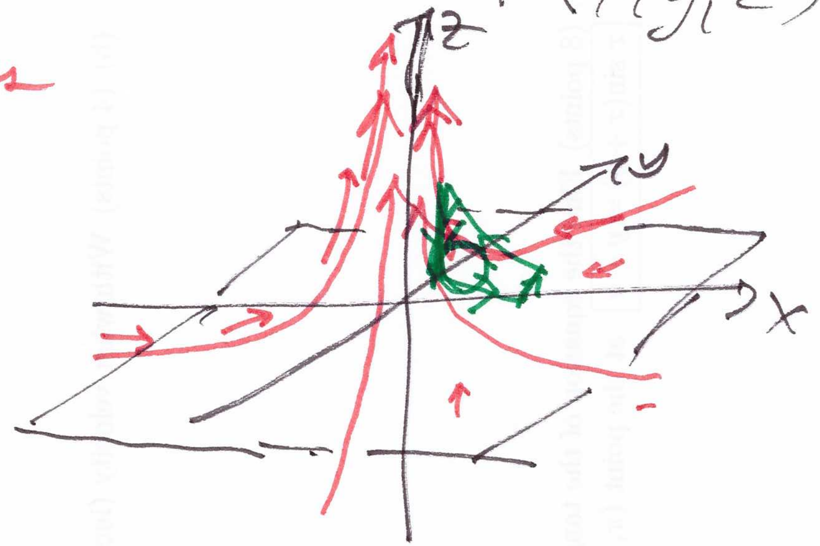
sphere $(\sqrt{1-z^2}\cos\theta, \sqrt{1-z^2}\sin\theta, z)$

$P(\theta, z) = ((1-z^2)\cos\theta, \sin\theta, z)$

$0 \le \theta \le 2\pi \qquad -1 \le z \le 1$

3-D vector field $\vec{F}(x,y,z) = \left(\frac{-x}{\sqrt{x^2+y^2}^2}, \frac{-y}{\sqrt{x^2+y^2}^2}, \frac{1}{x^2+y^2}\right)$?

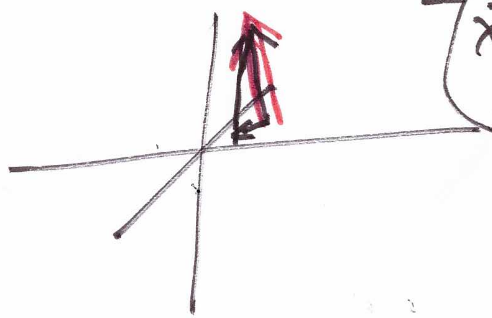
flow lines



$$= \frac{(-x, -y, 1)}{x^2+y^2}$$

$$= \frac{1}{x^2+y^2}(-x,-y,1)$$

f F



curl F ? $\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \times (p,q,r)$

compute curl $F = \vec{\nabla} \times \vec{F}$

$$\vec{\nabla} \times (f\vec{F}) \overset{!}{=} \nabla f \times \vec{F} + f(\nabla \times \vec{F}) = \text{vec}$$