A median of a triangle is the line segment from a vertex to the midpoint of the opposite side.

**Proposition:** Given any triangle in the plane, the three medians intersect at a common point which is \( \frac{2}{3} \) of the way along each median.

**Proof:** We translate the proposition into the language of vector algebra. Let \( \vec{u}, \vec{v}, \vec{w} \) be the vectors from the origin to the vertices of the triangle.

The vector from \( \vec{v} \) to \( \vec{w} \) is \( \vec{w} - \vec{v} \), and the midpoint of the corresponding side is:

\[
\vec{v} + \frac{1}{2}(\vec{w} - \vec{v}) = \frac{1}{2}\vec{v} + \frac{1}{2}\vec{w}.
\]

The vector from \( \vec{u} \) to this midpoint is \( \frac{1}{2}\vec{v} + \frac{1}{2}\vec{w} - \vec{u} \), and the median from \( \vec{u} \) to the midpoint is the parametrized line segment:

\[
\vec{\ell}(t) = \vec{u} + t\left(\frac{1}{2}\vec{v} + \frac{1}{2}\vec{w} - \vec{u}\right) \quad \text{for} \quad 0 \leq t \leq 1.
\]

(That is, the points of the segment are the endpoints of the vectors \( \vec{\ell}(t) \), written in standard form from the origin.) The point \( \frac{2}{3} \) of the way along this median is:

\[
\vec{\ell}\left(\frac{2}{3}\right) = \vec{u} + \frac{2}{3}\left(\frac{1}{2}\vec{v} + \frac{1}{2}\vec{w} - \vec{u}\right)
= \vec{u} + \frac{1}{3}\vec{v} + \frac{1}{3}\vec{w} - \frac{2}{3}\vec{u}
= (1 - \frac{2}{3})\vec{u} + \frac{1}{3}\vec{v} + \frac{1}{3}\vec{w}
= \frac{1}{3}(\vec{u} + \vec{v} + \vec{w}),
\]

where we expand and factor using the distributive property of scalar multiplication over vector addition.

The same computation for the other two medians gives their \( \frac{2}{3} \) points:

\[
\vec{u} + \frac{2}{3}\left(\frac{1}{2}\vec{u} + \frac{1}{2}\vec{w} - \vec{u}\right) = \frac{1}{3}(\vec{u} + \vec{v} + \vec{w}),
\]

\[
\vec{w} + \frac{2}{3}\left(\frac{1}{2}\vec{u} + \frac{1}{2}\vec{v} - \vec{w}\right) = \frac{1}{3}(\vec{u} + \vec{v} + \vec{w}).
\]

Thus, all three medians contain a common point, the \( \frac{2}{3} \)-point along each. \( \square \)

**Note:** We call this common point the **centroid** of the triangle. The proof shows it is the vector average of \( \vec{u}, \vec{v}, \vec{w} \).