Math 254H # Weekly Homework 7 **Due 10/14/2019**

For each double integral below: (a) sketch the region D defined by the bounds of integration; (b) reverse the order of integration; and (c) evaluate the integral in the most efficient way.

1. $\displaystyle\int_{x=0}^{1}\int_{y=x^3}^{x^2} y\,dy\,dx$ **2.** $\displaystyle\int_{x=0}^{1}\int_{y=1}^{e^x} y(x{+}y)\,dy\,dx$ **3.** $\displaystyle\int_{y=0}^{1}\int_{x=y}^{1} \sin(x^2)\,dx\,dy$

For each of the two solids below, write it in terms of a graph $z = f(x,y)$ over a domain D, and evaluate its volume using the polar substitution $P(r,\theta)$; if the domain is parametrized as $D = P(D^*)$, we have:

$$\iint_D f(x,y)\,dx\,dy \;=\; \iint_{D^*} f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta.$$

4. A right circular cone with base radius R and height h.

5. A sphere of radius R. *Hint:* Work with a hemisphere.

6. For the vector field $\vec{F}(x,y) = (x{+}y, xy)$, the unit disk D, and its boundary the counterclockwise unit circle $\mathbf{c}$, evaluate integrals to verify:

(a) *Curl Theorem:* The integral of the rate of circulation of $\vec{F}$ over the region is equal to the total circulation of $\vec{F}$ around the region:

$$\iint_D \operatorname{curl}\vec{F}(x,y)\,dx\,dy \;=\; \oint_{\mathbf{c}} \vec{F}(\mathbf{c})\cdot d\mathbf{c}.$$

(b) *Divergence Theorem:* The integral of the rate of flux of $\vec{F}$ over the region is equal to the total flux of $\vec{F}$ out of the region:

$$\iint_D \operatorname{div}\vec{F}(x,y)\,dx\,dy \;=\; \oint_{\mathbf{c}} \vec{F}(\mathbf{c})\cdot d\mathbf{n}.$$