

Summary of the kinds of functions we have covered, how to visualize them, their derivatives and integrals.

function	picture	approximation	derivative	integral theorem
$f : \mathbb{R} \rightarrow \mathbb{R}$ $f(x)$	graph $y = f(x)$ derivative vect field	$f(x) \approx f(x_o) + f'(x_o)(x - x_o)$	tangent slope $[Df_{x_o}] = [f'(x_o)]$	<i>Second Fund Thm of Calc</i> $\int_a^b f'(x) dx = f(b) - f(a)$
$f : \mathbb{R}^2 \rightarrow \mathbb{R}$ $f(x, y)$	graph $z = f(x, y)$ contour map $f(x, y) = c$ gradient vector field ∇f	$f(x, y) \approx f(x_o, y_o) + \nabla f(x_o, y_o) \cdot (x - x_o, y - y_o)$	gradient vector $[\nabla f(x_o, y_o)] = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{bmatrix}$	<i>Gradient Theorem</i> $\int \nabla f(\vec{c}) \cdot d\vec{c} = f(\vec{c}(1)) - f(\vec{c}(0))$
$\vec{c} : \mathbb{R} \rightarrow \mathbb{R}^2$ $\vec{c}(t) = (x(t), y(t))$ $a \leq t \leq b$	parametrized curve	$\vec{c}(t) \approx \vec{c}(t_o) + \vec{c}'(t_o)(t - t_o)$	tangent vector $[\vec{c}'(t_o)] = \begin{bmatrix} x'(t_o) \\ y'(t_o) \end{bmatrix}$	<i>Length Formula</i> $\int_a^b \vec{c}'(t) dt = \text{len}(\vec{c})$
$G : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ $G(u, v) = (x(u, v), y(u, v))$	uv -grid in xy -plane parametrized region $D = G(D^*)$	$G(u, v) \approx G(u_o, v_o) + DG_{(u_o, v_o)}(u - u_o, v - v_o)$	Jacobian matrix $[DG_{(u_o, v_o)}] = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix}$	<i>Substitution Formula</i> $\iint_D f(x, y) dx dy = \iint_{D^*} f(G(u, v)) \det DG du dv$
$\vec{F} : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ $\vec{F}(x, y) = (p(x, y), q(x, y))$	field of arrows in xy -plane		rate of circulation $\text{curl } \vec{F} = \frac{\partial q}{\partial x} - \frac{\partial p}{\partial y}$ rate of flux $\text{div } \vec{F} = \frac{\partial p}{\partial x} + \frac{\partial q}{\partial y}$	<i>Curl Theorem</i> $\iint_D \text{curl } \vec{F}(x, y) dx dy = \oint \vec{F}(\vec{c}) \cdot d\vec{c}$ <i>Divergence Theorem</i> $\iint_D \text{div } \vec{F}(x, y) dx dy = \oint \vec{F}(\vec{c}) \cdot d\vec{n}$ $\vec{c} = \text{ctr-clock boundary of } D$ $d\vec{c} = (x'(t), y'(t)) dt$ $\vec{n} = \text{outward normal of } D$ $d\vec{n} = (y'(t), -x'(t)) dt$