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Math 254H **Quiz 18** **Feb 22, 2017**

Recall that $F : \mathbb{R}^n \to \mathbb{R}^m$ has Jacobian derivative matrix $[DF]$ with m rows, n columns.

1. Consider a function $\mathbf{c} : \mathbb{R} \to \mathbb{R}^2$, tracing the curve $\mathbf{c}(t) = (x(t), y(t))$. Write the derivative matrix $[D\mathbf{c}_t]$ in terms of the coordinate functions $x(t)$ and $y(t)$.

Solution: Since the function has 2-dimensional output, 1-dimensional input, the derivative matrix has 2 rows, 1 column:

$$[D\mathbf{c}_t] \;=\; [\mathbf{c}'(t)] \;=\; \begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix}.$$

This is just the matrix form of the tangent velocity vector $\mathbf{c}'(t)$.

2. Consider a function $f : \mathbb{R}^2 \to \mathbb{R}$. Write the derivative matrix $[Df_{(x,y)}]$.

Solution: Since the function has 1-dimensional output, 2-dimensional input, the derivative matrix has 1 rows, 2 columns:

$$[Df_{(x,y)}] \;=\; [\nabla f(x,y)] \;=\; \begin{bmatrix} \frac{\partial f}{\partial x} \,,\, \frac{\partial f}{\partial y} \end{bmatrix}.$$

3. Use the Chain Rule to compute the derivative matrix of the composite function $F : \mathbb{R}^2 \to \mathbb{R}^2$ given by $F(u,v) = \mathbf{c}(f(u,v))$.

Solution: The Chain Rule says the derivative of the composite function is the composite of the derivative linear functions. That is, the Jacobian matrix of the composite is the matrix product of individual Jacobians, with dimensions $(2 \times 1) \cdot (1 \times 2) = (2 \times 2)$:

$$[DF_{(u,v)}] \;=\; [D(\mathbf{c} \circ f)_{(u,v)}] \;=\; [D\mathbf{c}_{f(u,v)}] \cdot [Df_{(u,v)}]$$

$$= \begin{bmatrix} x'(f(u,v)) \\ y'(f(u,v)) \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial f}{\partial x} \,,\, \frac{\partial f}{\partial y} \end{bmatrix}. \;=\; \begin{bmatrix} x'(f(u,v))\, \frac{\partial f}{\partial x} & x'(f(u,v))\, \frac{\partial f}{\partial y} \\ y'(f(u,v))\, \frac{\partial f}{\partial x} & y'(f(u,v))\, \frac{\partial f}{\partial y} \end{bmatrix}$$