# Math 254H 3/20/2020

Trouble with WHW schedule?

Recommend LaTeX

or single scanned picture file

$$\boxed{\begin{array}{l}\vec{F}: \mathbb{R}^3 \to \mathbb{R}^3 \\ \text{vector field} \\ \vec{F} = (p, q, r)\end{array}}$$

Conservative $\vec{F} = \nabla f$

$\Updownarrow$

Path indep, circulation-free

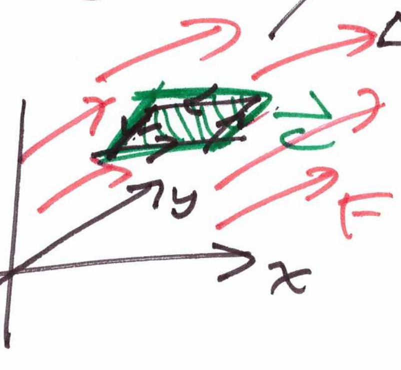
$\oint \vec{F}(\vec{c}) \cdot d\vec{c} = 0$

closed $\vec{c}$

curl Thm $\Uparrow$ $\Downarrow$ Def of curl

Irrotational $\text{curl}\,\vec{F} = 0$

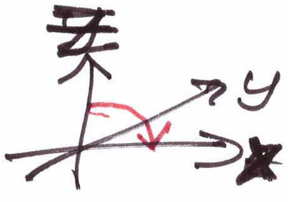
$$\text{curl}_{xy} \vec{F} = \lim_{\substack{\vec{c} \to (x,y) \\ \Delta x, \Delta y \to 0}} \int_{\vec{c}} \vec{F}(\vec{c}) \cdot d\vec{c} \Big/ \underbrace{\text{area}(\vec{c})}_{\Delta x \Delta y} = \frac{\partial p}{\partial x} - \frac{\partial q}{\partial y}$$




$\text{curl}\,\vec{F} =$

$$\Big(\underbrace{\text{curl}_{yz}\vec{F}}_{x\text{-axis}}, \underbrace{-\text{curl}_{xz}\vec{F}}_{y\text{-axis}}, \underbrace{\text{curl}_{xy}\vec{F}}_{z\text{-axis}}\Big)$$

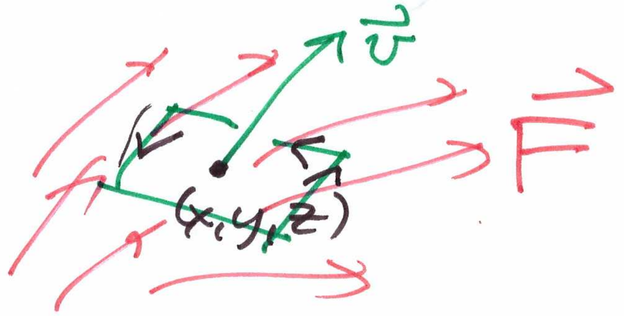
$$-\text{curl}_{xz}\vec{F} = \text{curl}_{\overrightarrow{zx}}\vec{F}$$



directional curl

$\text{curl}_{\vec{v}} \vec{F}$ = rate of circulation of $\vec{F}$
in plane perp to $\vec{v}$
near $(x,y,z)$

$|\vec{v}| = 1$




$$\text{curl}_{\vec{v}} \vec{F} = (\text{curl}\, \vec{F}) \cdot \vec{v}$$

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Why does circ-free $\oint \vec{F}(\vec{c}) \cdot d\vec{c} = 0$

imply $\text{curl}\, \vec{F} = 0$ ? ⇙ def of curl
irrotational?

$\|$

$$\left(\text{curl}_{yz} \vec{F}, -\text{curl}_{xz} \vec{F}, \text{curl}_{xy} \vec{F}\right)$$

$\|$

$$\lim \frac{\oint \cancelto{0}{\vec{F}(\vec{c}) \cdot d\vec{c}}}{\text{area } \vec{c}}$$

If circulation is zero,
then curl = rate of circulation $\stackrel{!}{=} 0$

③

If $\vec{F}$ irrotational $\text{curl}\,\vec{F} = \vec{0}$
rate of circulation is zero

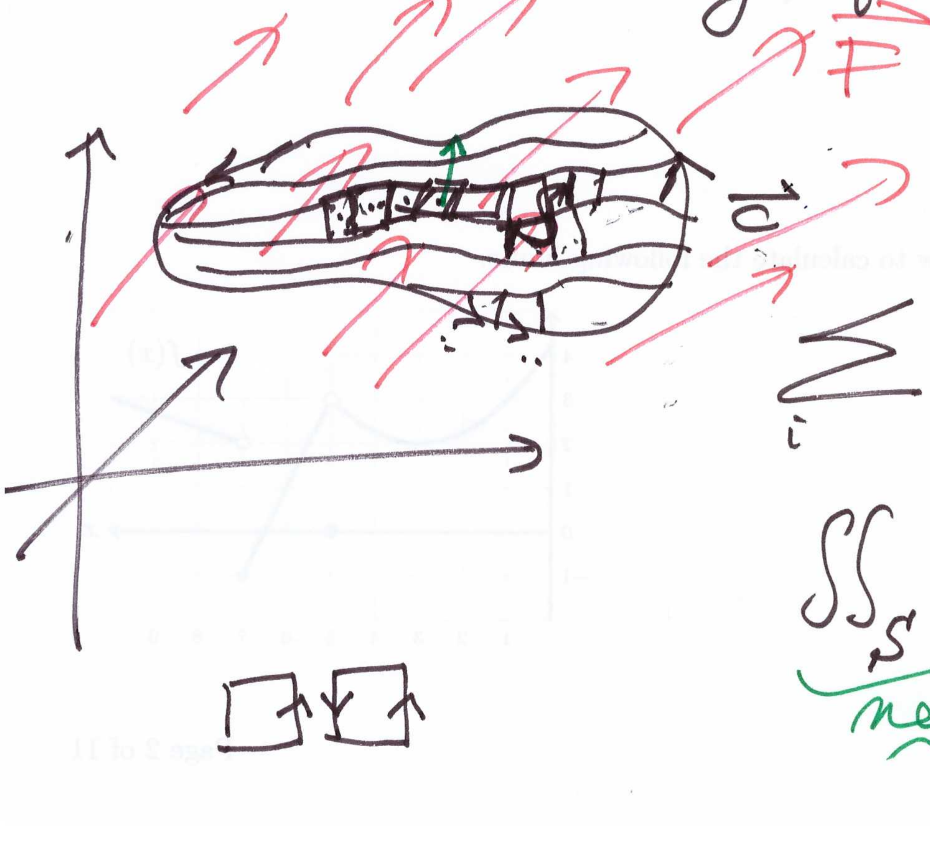
then why is $\vec{F}$ circulation free?

$$\oint \vec{F}(\vec{c}) \cdot d\vec{c} \stackrel{?}{=} 0 \quad \text{any closed } \vec{c}\,?$$

circulation is zero?

Curl Thm: Integral of rate of
(3D) circulation of $\vec{F}$.
over any surface

= total circulation around
boundary curve of surface




$$\oint \underline{\vec{F}(\vec{c})} \cdot d\vec{c}$$

$$\| $$

$$\sum_i \oint \vec{F}(\vec{c}_i) \cdot d\vec{c}_i$$

$$\| $$

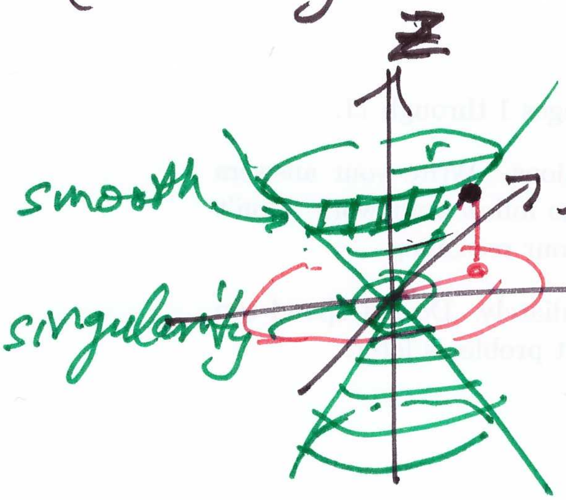
$$\iint_S (\underline{\text{curl}\,\vec{F}}) \cdot \vec{n}\, dS$$

need to define

# Understand surfaces in $\mathbb{R}^3$

Cut out by equations

$$\left\{ (x,y,z) \in \mathbb{R}^3 \;\middle|\; x^2+y^2 = z^2 \right\} = S$$




$$z = \underbrace{\sqrt{x^2+y^2}}_{r}$$

height

dist from z-axis $r$

## Parametrize

$$G : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$$

$$(u,v) \longrightarrow (x(u,v), y(u,v), z(u,v))$$

parameters    points of surface

## Coordinate system:

$$H : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$$

coord mapping $(u,v,w) \longrightarrow (x,y,z)$

cylindrical coordinates

$$\text{Cyl}: (r,\theta,z) \longrightarrow (r\cos\theta, r\sin\theta, z)$$

Parametrize cone:

$$G(r,\theta) = (r\cos\theta, r\sin\theta, \underbrace{r}_{z=r})$$

$$r \geq 0,\quad 0 \leq \theta \leq 2\pi$$

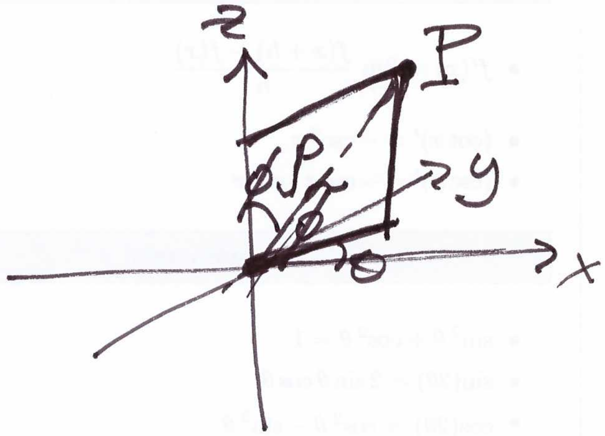
# Spherical coordinates

Define points in $\mathbb{R}^3$

by "latitude", "longitude", altitude

$(\theta \,,\, \phi \,,\, \rho)$

"rho" $\rho$




$\rho$ = dist from $\vec{0}$

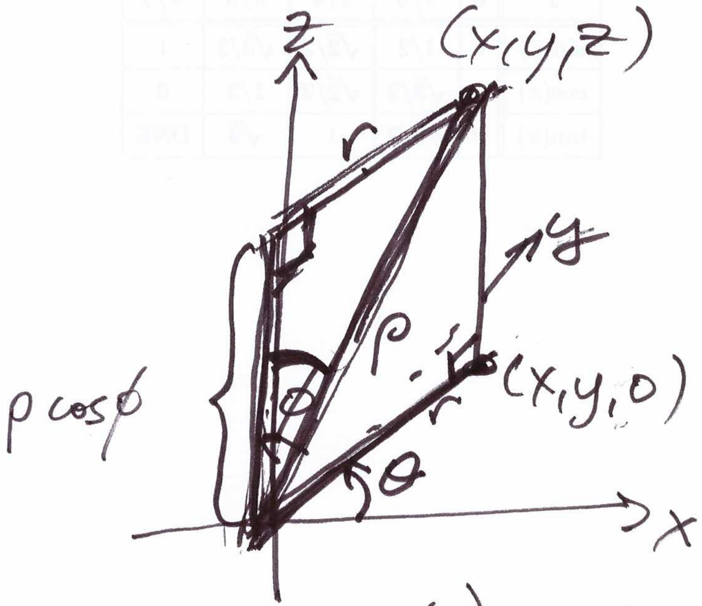
~~$\tilde{\phi}$ = tilt from xy-plane usual "latitude"~~

$\phi$ = tilt from z-axis

$\frac{\pi}{2} - \tilde{\phi}$

$90^\circ$ − latitude

$\theta$ = turn from x-axis in xy-plane




Sph: $(\rho, \theta, \phi) \longrightarrow (x, y, z)$

$z = \rho \cos\phi$ $\qquad \cos\phi = \frac{z}{\rho}$

$r = \rho \sin\phi$ $\qquad \sin\phi = \frac{r}{\rho}$

$x = r\cos\theta = \rho \sin\phi \cos\theta$

$y = r\sin\theta = \rho \sin\phi \sin\theta$

$Sph(\rho, \theta, \phi) = (\rho \sin\phi \cos\theta, \rho \sin\phi \sin\theta, \rho\cos\phi)$

radius, turn, tilt

Parametrize sphere $\rho = \text{const } \rho_0$

$Sph(\rho_0, \underline{\theta, \phi})$