Additions/Corrections

Calculus of Variations

C. R. MacCluer, Prentice Hall 2005

Chapter 1

Chapter 2

Exercise 2.35 You and the other traffic are maintaining a constant speed in the right lane of a freeway. You wish to glance ahead by momentarily moving into the left lane. What constant-speed strategy will cost you the least headway? Should you move quickly into the left lane then back, or should you change lanes slowly? Formulate this problem.

Chapter 3

Chapter 4

Exercise 4.54 Contrary to popular belief, the period T of oscillation of the planar pendulum (CVP 8) is not independent of its maximal angle θ_0 of excursion. Larger θ_0 yield larger periods T; in the extreme case, $T = \infty$ when $\theta_0 = \pi$.

Prove that

$$\frac{T}{4} = \sqrt{\frac{a}{2g}} \int_0^{\theta_0} \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}} = k\sqrt{\frac{a}{g}} \int_0^{\theta_0/2} \frac{d\phi}{\sqrt{1 - k^2 \sin^2\phi}},$$

where $k = \sin(\theta_0/2)$. Prove that in the limit as $\theta_0 \to 0$, the period T approaches the period $2\pi\sqrt{a/g}$ of the harmonic oscillator $\ddot{\theta} = -(g/a)\theta$.

Chapter 5

Exercise 5.39 We have seen paths that are stationary for a cost but do not minimize it — see (3.9). Attempt to formulate the notion that a path $y = y(x)$ *makes stationary* an integral cost like (5.1a) yet satisfies an integral constraint like (5.1b).

Chapter 6

Chapter 7

Chapter 8

Chapter 9

(Daniel Liberzon) The first term in the integrand of (9.46) should be $L_{y'y'}\eta'^2$

Exercise 9.15 typo: $x^T Ax \geq \gamma|x|^2 = \gamma\, x^T x$.

Exercise 9.45 The concepts *weak global minimum, strong global minimum,* and *minimum* coincide.

Chapter 10

Chapter 11

(John Prussing) Corollary B. (Jacobi's necessary condition)

(Daniel Liberzon) Last term in (11.16) should be $-(w-z)L_{y'}(x,y,z)$

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