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To the Student

This book takes several radical departures that will alter your method of study. First is its *brevity* — major concepts are presented with minimal possible detail. Details are pushed into the exercises, omitted, or postponed until later sections. This will require a more active role from you. I encourage you to consult other texts, fellow students, scientists, engineers, and mathematicians. Since this subject touches all the physical sciences and engineering, you will find a warm reception to your questions.

The second departure is the use of *Big Tools.* Rather than using ad hoc arguments, I have instead designated certain sophisticated mathematical tools as unproven *Principles*, thereafter to be used at will. This will enable us to proceed more cleanly and at an advanced level. You may need to spend some time thinking about the big picture.

Third is the highly physical orientation of this book. This Mathematics arose from problems in heat transfer and vibration, and is best understood in terms of these notions and language. Specific real-world applications are discussed. You may need to consult others for help with some physical details and intuition.

The fourth is the integration of *numerical methods* and *symbolic manipulation.* You may need to review MatLab or your favorite compiler or spreadsheet.

Fifth, this book is based on the modern *Sobolev methods.* This viewpoint is elegant and consonant with implementation by digital computer. This will require you to rethink the notions of limit and convergence.

The first 5 sections form an informal introduction that develop your physical and mathematical intuition. The next section introduces you to Hilbert space where this subject naturally belongs. In the following 6 sections, we pose and solve the standard problems of the subject. The last 7 sections are short introductions to selected topics.

I am interested in your comments and suggestions. I pay a finder's fee for typos. Email me at the address *maccluer@math.msu.edu*

To the Instructor

Two sections follow: *How this Book Differs* and *Using this Book.*

How this Book Differs

This book reflects three fundamental 'recent' shifts in paradigm — to the digital computer, to norm-based methods, and to Sobolev methods.

This book de-emphasizes pointwise convergence, the least useful and most pathological of all notions of convergence. Instead, deviations in shapes, signals, or temperature regimes are most naturally measured by their L^2 or L^1 norms, since these norms are physically natural measurements of power or energy. With a norm viewpoint, L^2, L^1, and uniform convergence can all be discussed in a simple and unified way. The Galerkin/Sobolev weak solutions are, after all, the practical computer implementable solutions.

In this book, the classical boundary and initial value problems are restated in a form more consistent with modern Dynamical Systems. PDEs are rewritten as linear 'ordinary' differential equations with constant (time-invariant) spatial differential operator coefficients; solutions are trajectories of undulating shapes, evolving from an initial shape over time, located with time varying coordinates with respect to a fixed and physically natural basis — the spatial eigenmodes. Solutions are moving points in Hilbert space.

A second departure from the standard texts is the use of *Big Tools*. The older texts attempt internal completeness at the expense of many ad hoc arguments. In this book, certain sophisticated tools are designated as *Principles*, thereafter to be used at will. These tools are referenced but not proven, stated in a way comprehensible to the intended audience. For example, completeness of the classical orthogonal expansions is deduced in all cases from Rellich's Theorem on the (compactness of the resolvent of the) Laplacian with zero boundary conditions. The approach is mature, elegant, and promotes interest in advanced course work.

Attempting internal completeness in a text at this level is a mistake. The Comap/Exxon survey shows that the bulk of advanced Mathematics is being taught outside mathematics departments. I suspect this is because we mathematicians insist on teaching what our clients do not need nor want to know. Our clients need powerful mathematical tools explained clearly. They are not interested in the inner details or pathologies at the fringe of the subject. Such a passion for internal completeness would make grade-school arithmetic inaccessible. So it is with BVPs. Older BVP texts assume the student exists in a vacuum, without background, experience, or resources. This text encourages students to consult with mathematicians, engineers, fellow students, and to visit the library.

A third departure is the highly physical orientation of this book. Such an approach appeals to the clientele of such a course and fixes ideas well. Dimensionless variables are employed yielding clean, detail-free problems. One may think of this use of the physical notions of heat flux, temperature, voltage, etc., as an alternative language, just as in algebraic geometry the geometric language is a means for remembering the essentially algebraic results.

I have taken a further step. Not only are problems often discussed in physical language, but specific real-world applications are described. For example, the Kelvin Line Source problem is motivated by a discussion of the seasonal temperature variations about the vertical heat exchanger of a ground-coupled heat pump. There will be descriptions of problems stemming from large flexible structures and structural damping, terminated ideal and lossy transmission lines, waveguides, acoustics, Schroedinger's hydrogen atom, topics in building technology, control of distributed systems, heat exchangers, ELF submarine communication, RF currents on a conductor, cavity resonators, variations in subsurface ground temperatures, diffusion of pollutants into an environment, cooling of curing composite materials, petroleum exploration, etc. It is important with the modern student to appear directly relevant. It is no longer enough to motivate a separation of variables problem with an off-handed, "this comes from a problem in heat transfer."

A fifth departure is to tacitly base the book on the modern Sobolev viewpoint. Ultimately all the theory rests on compact embeddings and weak solutions. This is not explicit until the last two chapters.

The intended audience is senior or first-year graduate students of applied mathematics, chemistry, physics, electrical, mechanical, or civil engineering. The prerequisite background is a solid grounding in linear algebra and a first course in ordinary differential equations.

Because simulation has largely replaced experimentation, the modern graduate engineering education is becoming far more analytical. Many beginning graduate engineering students, wishing to fill gaps in their background, will find this modern treatment ideal.

The thrust of this book is to present linear BV Problems in a unified, simple, yet modern fashion. This is not an encyclopedia of BVP solutions. The student, once armed with the solid foundation provided by this book, can intelligently search the famous collections for solutions to a particular application. The book is short, concise, and is limited to a short list of standard representative problems that are reprised again and again throughout the book.

Using this Book

This book can be used in standard lecture format or in various degrees of the R. L. Moore system. Many if not most details of the development are given as exercises. These exercises can be worked and presented in lecture for a continuous flow, or assigned for homework in some mix consonant with your own style. Other exercises are routine variations of the worked examples.

The first 5 sections form an informal and intuition-building introduction to boundary value problems. Nothing from Chapters 1–4 is truly essential. However all of Chapter 5 is essential material that cannot be omitted.

Chapter 6 introduces the mathematical tools: norms and inner products, orthogonal series. The essential ideas are orthogonal bases, eigenfunctions, and Rellich's theorem. Much of this theoretical material could be passed over, although this would be a disservice to the young scientist or engineer. The crux of the book (the following 6 sections) does not depend heavily on this theoretical material.

The next 6 chapters (7–12) consist of three pairs of chapters: Fourier series applied to rectangular problems, Bessel functions applied to cylindrical problems, and orthogonal polynomials applied to spherical problems. Conceivably a course could be taught using Chapter 5 and these 6 Chapters alone.

The last 7 chapters are short introductions to selected topics. The underpinning of the book in the Sobolev viewpoint is finally revealed in the last two sections. Chapters 13, 14, and 19 are best taught to stronger students.

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