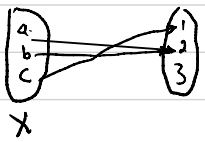
HW 1 wed

Maps and Functions:

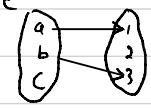
Let X and Y be sets. A function or map assigns a unique element of Y to every member of X; denoted $f: X \to Y$

ex:

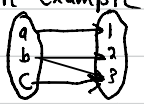


X

non-example



non-example



$f(x) = y$ or $f: x \overset{\text{maps to}}{\mapsto} y$. We say x maps to y under f.

def: If $f: X \to Y$ is a function we call X the source or domain and We call Y the target (or codomain)

ex: define $f: \mathbb{Z} \to \mathbb{Z}$ by $f: x \mapsto x^2$

define $f: \mathbb{N} \to \mathbb{R}^+$ by $f: x \mapsto \sqrt{x}$

Let $X = \{Y \text{ is a set} : |Y| < \infty\}$ then $|\cdot|$ is a function.

$|\cdot|: X \to \mathbb{N}$

non-example

$f: \mathbb{R} \to \mathbb{R}$ defined by $f: x \mapsto \frac{x}{x-1}$. (undefined for $x = 1$)

corrected

Let $X := \{x \in \mathbb{R} : x \neq 1\}$. Then,

$f: X \to \mathbb{R}$ defined by $f: x \mapsto \frac{x}{x-1}$ is a function

Writing Tips

The responsibility of communication lies with you. If someone at your level can't understand what you have written, Then the problem is your writing

- Write in Sentances and Paragraphs,
- Equals means equals
  - non-ex: Factors: $6 = 2$ and 3
  - correction: The factors of 6 are 2 and 3.
- If you use 'if', Then use 'then'
  - non-ex: If x is odd, x^2 is odd
  - correction: If x is odd, Then x^2 is odd.
- Call things by their correct names
  - $6x^2+2$ (expression)
  - $3x^2+6 = 4x$ (equation)
  - $D = \sqrt{b^2-4ac}$ (formula)
- Avoid it
- Do not start Sentances with numbers, Symbols, or other objects.

Theorem: Let A and B be sets. Then $A \cap B \subseteq B \subseteq A \cup B$.

Pf

Let x be an arbitrary element of $A \cap B$. That is, $x \in A \cap B$. Then, by the definition of an intersection of sets $x \in A$ and $x \in B$. Thus, since x was arbitrary, every element of $A \cap B$ is in B and so $A \cap B \subseteq B$. Next, let y be an arbitrary element in B. That is, $y \in B$. Then, by the definition of the union of sets, $y \in A \cup B$. Thus, since y was arbitrary, every element of B is in $A \cup B$ and so $B \subseteq A \cup B$. Hence, $A \cap B \subseteq B \subseteq A \cup B$. □

Theorem: $X = Y$ if and only if $X \subseteq Y$ and $Y \subseteq X$

Pf First, we show that if $X = Y$ then $X \subseteq Y$ and $Y \subseteq X$.

Let x be an arbitrary element of X; That is, $x \in X$. Then, by the definition of the equality of sets $x \in Y$. Since x was arbitrary every member of X is a member of Y. Second, Let y be an arbitrary element of Y. Then, by the definition of equlity of sets once more, $y \in X$. Thus, since y was arbitrary $Y \subseteq X$

Next, we show if $X \subseteq Y$ and $Y \subseteq X$ Then $X = Y$.

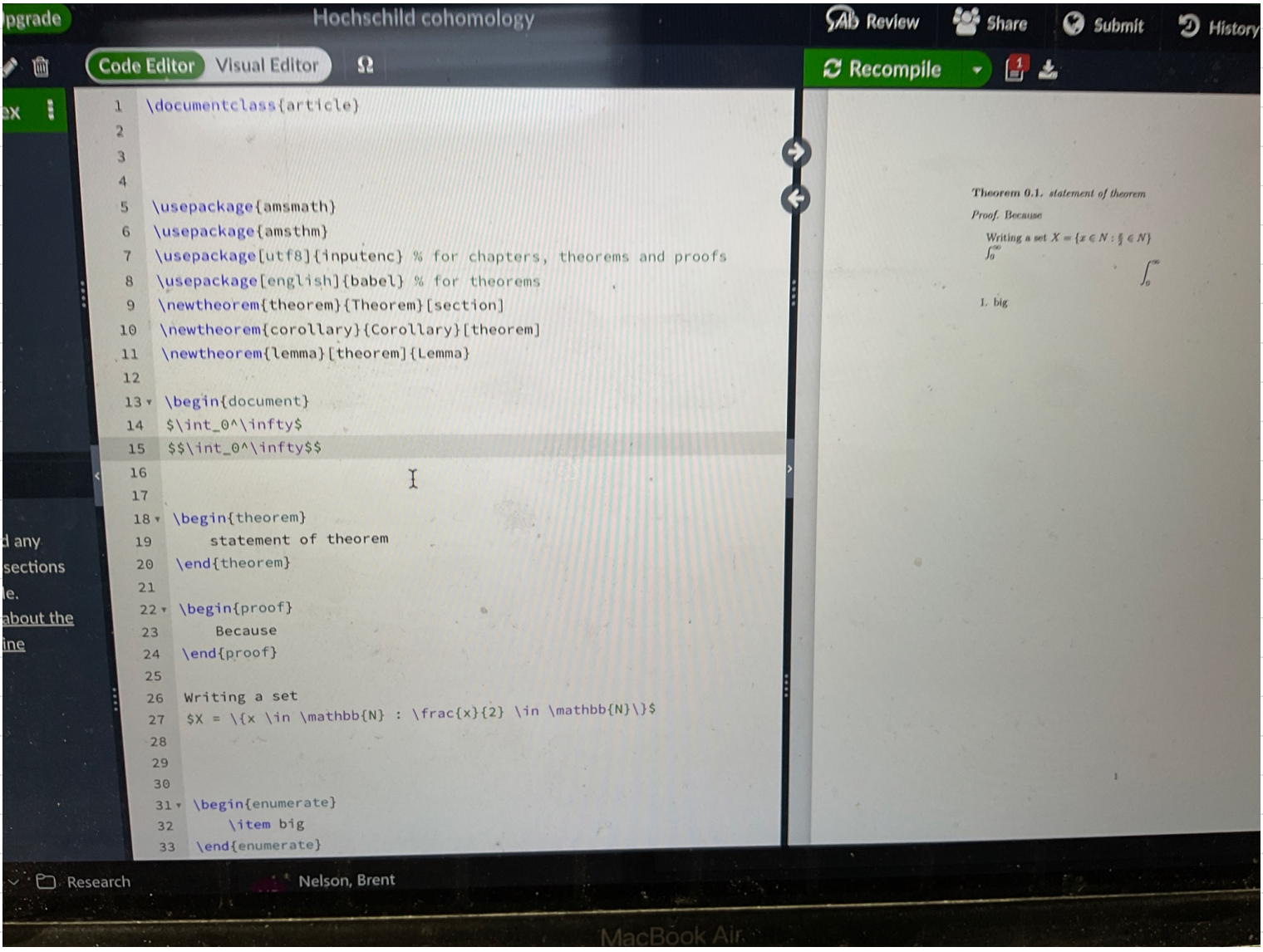
Since $X \subseteq Y$ every member of X is a member of Y and

Since $Y \subseteq X$ every member of Y is a member of X.

Thus, $(x \in X \Rightarrow x \in Y)$ and $(y \in Y \Rightarrow y \in X)$ and so $(x \in X \Leftrightarrow x \in Y)$ which is the definition of equality of sets. □

Claim: Let X and Y be finite sets. If $Y \subseteq X$ Then $|Y| \leq |X|$.

Pf Since Y is finite we may enumerate its elements, Say $Y = \{y_1, \cdots, y_n\}$ for some $n \in \mathbb{Z}^+$ so that $|Y| = n$. Next, Since X is finite we may enumerate its elements also, Say $X = \{x_1, \cdots, x_m\}$ for some $m \in \mathbb{Z}^+$ Then since $Y \subseteq X$, by rearranging and possibly relabeling we may set $y_i = x_i$ for $i \in \{1, \cdots, n\}$. Hence, we may write $X = \{y_1, \cdots, y_n, x_{n+1}, \cdots x_{n+k}\}$ for some $k \in \mathbb{Z}^+$. Thus, $|X| = n+k$. Additionally, since $k \geq 0$ it follows that $n \leq n+k$. Thus, $|Y| \leq |X|$. □


Hochschild cohomology
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\documentclass{article}
\usepackage{amsmath}
\usepackage{amsthm}
\usepackage[utf8]{inputenc} % for chapters, theorems and proofs
\usepackage[english]{babel} % for theorems
\newtheorem{theorem}{Theorem}[section]
\newtheorem{corollary}{Corollary}[theorem]
\newtheorem{lemma}[theorem]{Lemma}
\begin{document}
$\int_0^\infty$
$$\int_0^\infty$$
\begin{theorem}
statement of theorem
\end{theorem}
\begin{proof}
Because
\end{proof}
Writing a set
$X = \{x \in \mathbb{N} : \frac{x}{2} \in \mathbb{N}\}$
\begin{enumerate}
\item big
\end{enumerate}
Theorem 0.1. statement of theorem
Proof. Because
Writing a set $X = \{x \in N : \frac{x}{2} \in N\}$
$\int_0^\infty$
$\int_0^\infty$
1. big
Research
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