4 Using The Derivative

4.1 Local Maxima and Minima

* Local Maxima and Minima

Suppose p is a point in the domain of f:

- f has a **local minimum** at p if $f(p)$ is *less than or equal to* the values of f for points near p.
- f has a **local maximum** at p if $f(p)$ is *greater than or equal to* the values of f for points near p.

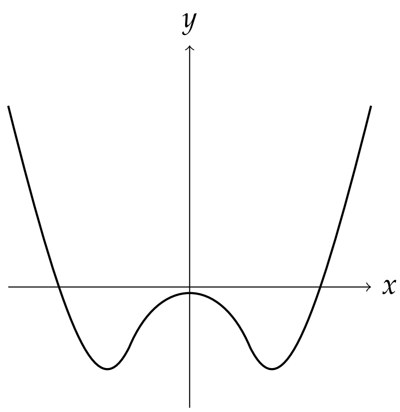
* How Do We Detect a Local Maximum or Minimum?

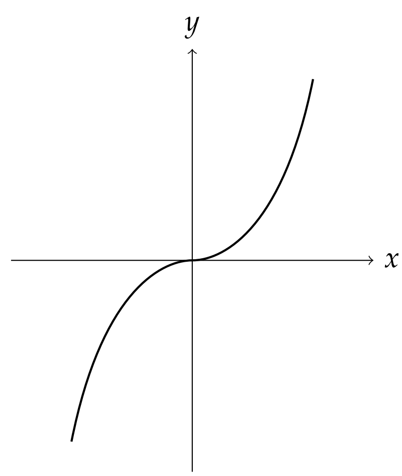
Suppose p is a point in the domain of f:

For any function f, a point p in the domain of f where $f'(p) = 0$ or $f'(p)$ is undefined is called a **critical point** of the function.

If a function, continuous on an interval (its domain), has a local maximum or minimum at p, then p is a critical point or an endpoint of the interval.

Example 1 *For $f(x)$ given below, indicate all critical points of the function f. How many critical points are there? Identify each critical point as a local maximum, a local minimum, or neither.*





Example 2 *(a) Graph a function with two local minima and one local maximum.*

(b) Graph a function with two critical points. One of these critical points should be a local minimum, and the other should be neither a local maximum nor a local minimum.

*** Testing For Local Maxima and Minima**

First Derivative Test for Local Maxima and Minima
Suppose p is a critical point of a continuous function f. Then, as we go from left to right:

- If f changes from decreasing to increasing at p, then f has a local minimum at p.
- If f changes from increasing to decreasing at p, then f has a local maximum at p.

Example 3 *(a) Graph a function f with the following properties:*

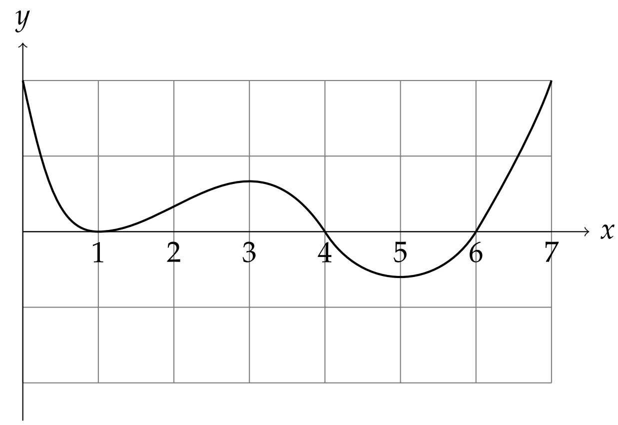
- *$f(x)$ has critical points at $x = 2$ and $x = 5$;*
- *$f'(x)$ is positive to the left of 2 and positive to the right of 5;*
- *$f'(x)$ is negative between 2 and 5.*

(b) Identify the critical points as local maxima, local minima, or neither.

Example 4 *Given the graph of $f'(x)$ below.*

(a) What are the critical points of the function $f(x)$?

(b) Identify each critical point as a local maximum, a local minimum, or neither.



Example 5 *Given $f(x) = x^3(1-x)^4$.*

(a) Find all critical points of f.

(b) Use the first derivative test to classify each critical point as a local max, a local min, or neither.

Second Derivative Test for Local Maxima and Minima
Suppose p is a critical point of a continuous function f, and $f'(p) = 0$.

- If f is concave up at p ($f''(p) > 0$), then f has a local minimum at p.
- If f is concave down at p ($f''(p) < 0$), then f has a local maximum at p.

Example 6 *Given* $f(x) = x^3 - 9x^2 - 48x + 52$.

(a) Find all critical points of f.

(b) Use the second derivative test to classify each critical point as a local max, a local min, or neither.

Example 7 *Find constants a and b so that* $f(x) = a(x - b\ln x)$ *has a local minimum at the point* $(2, 5)$.

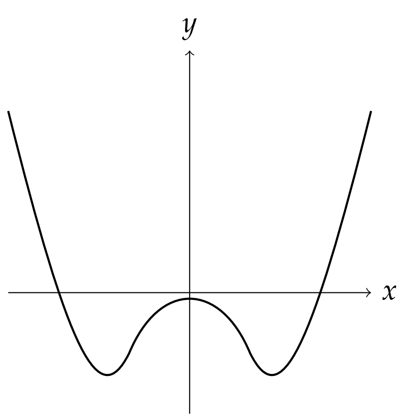
4.2 Inflection Points

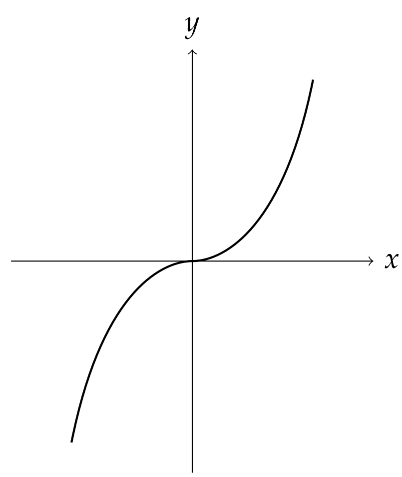
*** Concavity and Inflection Points**

A point at which the graph of a function f changes concavity is called an **inflection point** of f.

If p is an inflection point of f, then either $f''(p) = 0$ or f'' is undefined at p.

Example 1 *For the graph of $f(x)$ given below, indicate the approximate locations of all inflection points. How many inflection points are there?*





Example 2 *Find the inflection points of* $f(x) = x^3 - 9x^2 - 48x + 52$.

Example 3 *Find the critical points and inflection points of* $f(x) = xe^{-x}$.

Example 4 *Graph a function* f *with the following properties:* f *has a critical point at* $x = 4$ *and an inflection point at* $x = 8$*; the value of* f' *is negative to the left of 4 and positive to the right of 4; the value of* f'' *is positive to the left of 8 and negative to the right of 8.*

Example 5 *Sketch a graph of $y = f(x)$ such that*

(a) $f'(-1) = 0$, $f'(1) = 0$, $f'(3) = 0$

(b) $f'(x) > 0$ *for* $x < -1$ *and* $-1 < x < 3$

(c) $f'(x) < 0$ *for* $x > 3$

(d) $f''(-1) = 0$, $f''(1) = 0$

(e) $f''(x) < 0$ *for* $x < -1$ *and* $x > 1$

(f) $f''(x) > 0$ *for* $-1 < x < 1$