1.1 Linear and Rational Equations

In this section you will learn to:

- find restrictions on variable values
- solve linear equations in one variable
- solve rational equations with variables in the denominator
- recognize equations that are identities, conditional, or contradictions
- solve formulas for a specific value

Finding Variable Restrictions

Example 1: Find the restrictions on the variable a in each of the equations below.

(a) $\dfrac{2a}{a-5} = \dfrac{3}{2a+5}$

(b) $\dfrac{3}{a^2 - 3a - 4} = \dfrac{a-5}{a}$

Solving Linear Equations ($ax + b = 0$, where a and b are real numbers and $a \neq 0$)

Example 2: Solve $3(x-6) = 6x - x$

Steps: (if coefficients are integers)

1.

2.

3.

4.

Example 3: Solve $5x - (2x+2) = x + (3x-5)$

Example 4: Solve: $\frac{3x}{5} = \frac{2x}{3} + 1$ (NOTE: This equation is **equivalent** to $\frac{3}{5}x = \frac{2}{3}x + 1$.)

<u>Steps</u>: (if coefficients are fractions)

1.

2.

3 – 6.

<u>Solving Rational Equations</u> (equations having one or more "**rational**" expressions)

Example 5: Solve $\frac{5}{x} = \frac{10}{3x} + 4$

<u>Steps</u>: (if variable is in denominator)

1. Exclude any values that cause a zero denominator.
2. Find the LCD and multiply both sides by the LCD.
3. Simplify and solve the equation.
4. Check your solution in the **original** equation.

Example 6: Find all values of x for which $y_1 = y_2$ given $y_1 = \frac{22}{x^2 - 16}$ and $y_2 = \frac{1}{x+4} + \frac{1}{x-4}$.

Example 7: Solve for n: $\frac{1}{n-4}-\frac{5}{n+2}=\frac{6}{n^2-2n-8}$

Example 8: Solve: $\frac{x+2}{x+3}=1-\frac{1}{x^2+2x-3}$

Types of Equations		
Identity	**Conditional**	**Contradiction/Inconsistent**

Solving Formulas for a Specific Value

Steps:

1. If fractions are involved, multiply by LCD.
2. Move terms with desired variable to one side. Move all other terms to other side.
3. If two or more terms contain the desired variable, FACTOR out the variable.
4. Divide both sides by the “non variable” factor.

Example 9: Solve $V = \frac{1}{3}Bh$ for B.

Example 10: Solve $S = P + Prt$ for t.

Example 11: Solve $S = P + Prt$ for P.

Example 12: Solve $x = \frac{a-b}{c}$ for b.

Example 13: Solve $x = y + \frac{2w}{z}$ for z.

Example 14: Solve $C = \frac{A+B}{A-B}$ for A.

Example 15: Solve $\frac{1}{x} + \frac{2}{y} - \frac{3}{z} = 4$ for x.

1.1 Homework Problems

1. Find restrictions on the variable x in each equation below:

(a) $\frac{2}{x}=\frac{5}{x^2-4}$ (b) $\frac{x-3}{x(x+4)}=\frac{1}{x^2+3x-4}$ (c) $\frac{1}{3x^2}=\frac{x+3}{3x^3-27x}$

For Problems 2–8, solve the equations and classify each as an identity, contradiction or conditional.

2. $2x-4(5x+1)=3x+38$
3. $\frac{x}{2}-\frac{1}{2}-\frac{x}{5}=\frac{1}{10}$
4. $\frac{3x+1}{3}-\frac{1-x}{4}=\frac{13}{2}$
5. $14x+10=11(x+3)+3x$
6. $\frac{8}{x+2}+\frac{6}{x}=\frac{20}{x^2+2x}$
7. $7x-21=7(x-3)$
8. $\frac{x+2}{x+3}=1-\frac{1}{x^2+2x-3}$
9. Solve $C=2\pi\, r$ for r.
10. Solve $a=2b-3c$ for c.
11. Solve $\frac{a}{x}-\frac{y}{b}=3$ for x.
12. Solve $C=\frac{A+B}{A-B}$ for B.
13. Solve $A+B=\frac{AC+2D}{D}$ for D.
14. Solve: $\frac{1}{a}-\frac{2}{b}=\frac{3}{c}$ for b.
15. Solve $C=\frac{5}{9}(F-32)$ for F.

1.1 Homework Answers: 1.(a) $x\neq 0,\pm 2$ (b) $x\neq -4, 0, 1$ (c) $x\neq 0,\pm 3$ 2. {-2}; conditional 3. {2}; conditional 4. $\left\{\frac{77}{15}\right\}$; conditional 5. ϕ; contradiction 6. $\left\{\frac{4}{7}\right\}$; conditional 7. all real numbers; identity 8. {2}; conditional 9. $r=\frac{C}{2\pi}$ 10. $c=\frac{2b-a}{3}$ 11. $x=\frac{ab}{3b+y}$ 12. $B=\frac{AC-A}{C+1}$ 13. $D=\frac{AC}{A+B-2}$ 14. $b=\frac{2ac}{c-3a}$ 15. $F=\frac{9}{5}C+32$

1.2 Applications of Linear Equations

In this section you will learn to:

- use linear equations to solve word problems

Steps/Tips for Solving Word Problems:

1. **Read** the problem carefully. **Underline** key words and phrases. Let x (or any variable) represent one of the unknown quantities. **Draw** a picture or diagram if possible.
2. If necessary write expressions for **other unknowns** in terms of x.
3. Write a **verbal model** of the problem and then replace words with numbers, variables and/or symbols.
4. **Solve** the equation. **Answer** the question in the problem. **Label** answers!
5. **Check** your answer(s) in the original word problem (not in your equation).

Example 1: The number of cats in the U. S. exceeds the number of dogs by 7.5 million. The number of cats and dogs combined is 114.7 million. Determine the number of dogs and cats in the U. S.

Example 2: After a 30% discount, a cell phone sells for $112. Find the original price of the cell phone before the discount was applied to the purchase.

Example 3: Including a 6% sales tax, an item costs $91.69. Find the cost of the item before the sales tax was added.

Example 4: You are choosing between two car rental agencies. Avis charge $40/day plus $.10/mile to rent a car. Hertz charges $50/day plus $.08/mile. You plan to rent the car for three days. After how many miles of driving will the total cost for each agency be the same?

Example 5: The perimeter of a triangular lawn is 162 meters. The length of the first side is twice the length of the second side. The length of the third side is 6 meters shorter than three times the length of the second side. Find the dimensions of the triangle.

Example 6: (Simple Interest Problem) Tricia received an inheritance of $5500. She invested part of it at 8% simple interest and the remainder at 12% simple interest. At the end of the year she had earned $540. How much did Tricia invest at each interest amount?

Example 7: How many liters of a 9% solution of salt should be added to a 16% solution in order to obtain 350 liters of a 12% solution?

Example 8: A student scores 82%, 86% and 78% on her first three exams. What score is needed on the fourth exam for the student to have an average of 85% for all four exams?

Example 9: A student scores 85%, 72%, 96%, and 98% for his first four chapter exams. If the fifth exam, the final exam, counts twice as much as each of the chapter exams, is it possible for the student to get a high enough final exam score to get a 90% average for the course?

Example 10: A rectangular swimming pool measures 18 feet by 30 feet and is surrounded by a path of uniform width around all four sides. The perimeter of the rectangle formed by the pool and the surrounding path is 132 feet. Determine the width of the path.

Example 11: Sam can plow a parking lot in 45 minutes. Eric can plow the same parking lot in 30 minutes. If Sam and Eric work together, how long will it take them to clear the lot?

Example 12: A plane leaving Lansing International Airport travels due east at a rate of 500 mph. A second plane takes off 15 minutes later traveling in the same direction at 650 mph. How long will it take for the second plane to overtake the first?

1.2 Homework Problems

1. When a number is decreased by 30% of itself, the result is 56. What is the number?
2. $5\frac{1}{4}$% of what number is 12.6?
3. 25 is what % of 80?
4. Find .25% of $240.
5. After a 20% price reduction, a cell phone sold for $77. Find the original price of the phone.
6. Find two consecutive even integers such that the sum of twice the smaller integer plus the larger is 344.
7. The length of a rectangular garden plot is 6 feet less than triple the width. If the perimeter of the field is 340 feet, what are its dimensions?
8. Trinity College currently has an enrollment of 13,300 students with a projected enrollment increase of 1000 students a year. Brown college now has 26,800 students with a projected enrollment decline of 500 students per year. Based on these projections, when will the colleges have the same enrollment?
9. Sam invested $16,000 in two different stocks. The first stock showed a gain of 12% annual interest while the second stock suffered a 5% loss. If the total annual income from both investments was $1240, how much was invested at each rate?
10. How many liters of a 7% acid solution should be added to 30 liters of a 15% solution in order to obtain a 10% solution?
11. How many liters of skim milk (0% fat) must be added to 3 liters of milk containing 3.5% butterfat in order to dilute the milk to 2% butterfat?
12. Amy scored 78%, 64%, 98%, and 88% on her first four exams. What score does she need on her fifth exam in order to have an 85% average for all five exams?
13. Scott showed improvement on his five math exams throughout the semester improving by 3% on each successive exam. If the fifth exam (final exam) counted twice as much as the first four exams and his average was 79% for all five exams, what score did he receive on his first exam?
14. The length of a rectangular tennis court is 6 feet longer than twice the width. If the perimeter of the court is 228 feet, find the dimensions of the court.
15. During a road trip, Tony drove one-third the distance that Lana drove. Mark drove 24 more miles than Lana. The total distance they drove on the trip was 346 miles. How many miles did each person drive?
16. A garden hose can fill a swimming pool in 5 days. A larger hose can fill the pool in 3 days. How long will it take to fill the pool using both hoses?
17. The Smith family drove to their vacation home in Michigan in 5 hours. The trip home took only 3 hours since they averaged 26 mph more due to light traffic. How fast did they drive each way?

1.2 Homework Answers: 1. 80 2. 240 3. 31.25% 4. .6 5. $96.25 6. 114 and 116 7. 44 ft by 126 ft 8. 9 years 9. $12,000 @ 12%; $4000 @ 5% 10. 50 liters 11. 2.25 liters 12. 97% 13. 72% 14. 36 ft 78 ft 15. Tony: 46 miles; Lana: 138 miles; Mark:162 miles 16. 1.875 days (45 hrs) 17. 39 mph; 65 mph

1.3 Quadratic Equations

In this section you will learn to:

- solve quadratics equations by
  1. factoring
  2. square root property
  3. quadratic formula
  4. completing the square
  5. graphing (used mainly for checking – not considered an algebraic solution)
- use the discriminant to find the number and type of solutions (roots, x-intercepts, zeros)

A **quadratic equation** in x is an equation that can be written in the **general form**

$$ax^2 + bx + c = 0,$$

where a, b, and c are real numbers, with $a \neq 0$. A quadratic equation in x is also called a **second-degree polynomial equation.**

The Zero-Product Principle: If the product of two algebraic expressions is zero, then at least one of the factors equal to zero.

If $AB = 0$, then $A = 0$ or $B = 0$.

<u>Solving Quadratic Equations by Factoring</u>:

Example 1: $9x^2 = 12x$

Example 2: $x^2 = 3x + 10$

<u>Steps</u>:

1. Rearrange equation so that one side is 0.
2. Factor. (Use sum/product idea when a = 1. If $a \neq 1$, use grouping*.)
3. Set each factor equal to 0.
4. Solve each equation.
5. Check in original equation or by graphing (observe x-intercepts).

*Refer to Page 17 of Appendix A at the back of your textbook for steps using “Grouping Method”.

Example 3: $4x^2 - 13x = -3$ (Use "guess & check" or "grouping method".)

Solving Quadratic Equations by the Square Root Method:

Square Root Property: If $a > 0$ then $x^2 = a$ has two real roots: $x = \sqrt{a}$ or $x = -\sqrt{a}$

Reminder: Any time you choose to take a square root when solving an equation, you must include $\pm$.
(Example: If $x^2 = 4$, then $x = \pm 2$.)

Example 4: $3x^2 - 1 = 47$

Example 5: $(8x-3)^2 = 5$

Solving Quadratic Equations Using the Quadratic Formula:

Quadratic Formula: If $ax^2 + bx + c = 0$**,** where $a \neq 0$, then $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Example 6: Solve and simplify: $3x^2 = 5x - 1$

Example 7: Solve and simplify: $4x^2 + 16x = 13$

Example 8: Solve for t and simplify: $h = 32t - 16t^2$

In the quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the value of $b^2 - 4ac$ is called the **discriminant.**

Beware: The discriminant is **NOT** $\sqrt{b^2 - 4ac}$ **!!!**

$b^2 - 4ac > 0$	$b^2 - 4ac < 0$	$b^2 - 4ac = 0$

Example 9: Determine the number and type of solutions for the equations below. (Do not solve.)

(a) $2x - 3x^2 = 21$

(b) $25x^2 + 49 = 70x$

Solving a Quadratic Equation by Completing the Square:

Recall: Perfect Square Trinomials

$$(x+3)^2 = x^2 + 6x + 9$$

$$x^2 + 12x + 36 = (x+6)^2$$

$x^2 - 8x + \_\_\_ = (\qquad)^2$ $\qquad$ $x^2 + \_\_\_\_\_ + 4 = (\qquad)^2$ $\qquad$ $x^2 - \_\_\_\_\_ + 100 = (\qquad)^2$

Example 10: $2x^2 + 5x - 3 = 0$

Steps:

1. Divide each term by the leading coefficient a.

2. Move the constant term to the right side.

3. "Form" a perfect square trinomial by adding $\left(\frac{1}{2}b\right)^2$ to both sides.

4. Factor the left side (perfect square trinomial).
   Add the terms on the right side.

5. Finish solving using the Square Root Method.

1.3 Homework Problems

Solve Problems 1-6 by factoring:

1. $3x^2 = 5x$
2. $x^2 - 15 = 2x$
3. $-10x = x^2 + 25$
4. $a(a-12) - 15 = 30$
5. $2x^2 - 4x = 30$
6. $3m^2 = 7m + 6$

Solve Problems 7-9 by using the quadratic formula:

7. $x^2 + 15 = 8x$
8. $4x^2 - 8x + 1 = 0$
9. $4x^2 = 2x + 7$

Solve Problems 10-12 using the square root method:

10. $x^2 = \frac{25}{49}$
11. $(2x+11)^2 + 5 = 3$
12. $(3x-4)^2 = 8$

13. Solve for t: $h = 16t^2 - 4$

14. Solve for x: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

For Problems 15-17, determine the number and type of solutions by examining the discriminant.

15. $-3x^2 = 21 - 2x$
16. $2x^2 - 20x + 49 = 0$
17. $9x^2 + 49 = 42x$

Solve each of the following quadratic equations by completing the square:

18. $x^2 + 8x + 15 = 0$
19. $x^2 - 3x = 10$
20. $4x^2 = 7 - 8x$

1.3 Homework Answers: 1. $\left\{0\ \frac{5}{3}\right\}$ 2. {-3, 5} 3. {-5} 4. {-3, 15} 5. {-3, 5} 6. $\left\{-\frac{2}{3}, 3\right\}$ 7. {3, 5} 8. $\left\{\frac{2 \pm \sqrt{3}}{2}\right\}$ 9. $\left\{\frac{1 \pm \sqrt{29}}{4}\right\}$ 10. $\left\{\pm \frac{5}{7}\right\}$ 11. ϕ 12. $\left\{\frac{4 \pm 2\sqrt{2}}{3}\right\}$ 13. $\left\{\pm \frac{\sqrt{h+4}}{4}\right\}$ 14. $\left\{\pm \frac{a\sqrt{b^2 + y^2}}{b}\right\}$ 15. 0 real roots 16. 2 real roots 17. 1 real root 18. {-5, -3} 19. {-2, 5} 20. $\left\{\frac{-2 \pm \sqrt{11}}{2}\right\}$

1.4 Application of Quadratic Equations

In this section you will learn to:

- solve rational equations by changing to quadratic form
- use quadratic equations to solve word problems

Solving Rational Equations:

Example 1: $x-2=\dfrac{24}{x}$

Steps:

1. Multiply both sides by the LCD.
2. Simplify both sides.
3. Change to quadratic form:
   $ax^2+bx+c=0$.
4. Solve using a quadratic technique.
5. Check answer in original equation.
   (Be aware of restrictions!)

Example 2: $\dfrac{1}{x-1}+\dfrac{1}{x-4}=\dfrac{5}{4}$

Example 3: The length of a rectangle exceeds its width by 3 feet. If its area is 54 square feet, find its dimensions.

Example 4: The MSU football stadium currently has the 4^{th} largest HD video screen of any college stadium. The rectangular screen's length is 72 feet more than its height. If the video screen has an area of 5760 square feet, find the dimensions of the screen. (MSU math fact: The area of the video screen is about 600 ft^2 larger than Breslin's basketball floor.)

Example 5: When the sum of 8 and twice a positive number is subtracted from the square of the number, the result is 0. Find the number.

Example 6: Find at least two quadratic equations whose solution set is $\left\{-\frac{2}{3}, 5\right\}$.

Example 7: The height of an object thrown upward from the roof of a building 200 feet tall, with an initial velocity of 100 feet/second, is given by the equation $h = -16t^2 + 100t + 200$, where h represents the height of the object after t seconds. How long will it take the object to hit the ground?
(Round answer to nearest hundredth.)

Example 8: John drove his moped from Lansing to Detroit, a distance of 120 km. He drove 10 km per hour faster on the return trip, cutting one hour off of his time. How fast did he drive each way?

Example 9: In a round-robin tournament, each team is paired with every team once. The formula below models the number of games, N, that must be played in a tournament with x teams. If 55 games were played in a round-robin tournament, how many teams were entered?

$$N = \frac{x^2 - x}{2}$$

Example 10: When tickets for a rock concert cost \$15, the average attendance was 1200 people. Projections showed that for each 50¢ decrease in ticket prices, 40 more people would attend. How many attended the concert if the total revenue was \$17,280?

1.4 Homework Problems

1. Solve: $\frac{1}{x}=\frac{1}{3}-\frac{1}{x+2}$

2. Solve: $\frac{1}{x-1}=1-\frac{2}{x+1}$

3. The base of a triangle exceeds its height by 17 inches. If its area is 55 square inches, find the base and height of the triangle.

4. A regulation tennis court for a doubles match is laid out so that its length is 6 feet more than two times its width. The area of the doubles court is 2808 square feet. Find the length and width of a doubles court.

5. If 120 games were played in a round-robin tournament, how many teams were entered? (Refer to Example 9 in class notes for formula.)

6. A quadratic equation has two roots: ¾ and -5. (a) Find a quadratic equation where the coefficient of the x^2 term is 1. (b) Find a second equation that has only integers as coefficients.

7. The height of a toy rocket launched from the ground with an initial velocity of 128 feet/second, is given by the equation $h=-16t^2+128t$, where h represents the height of the rocket after t seconds. How long will it take the rocket to hit the ground? (Round answer to nearest hundredth.)

8. The height of an object thrown upward from the roof of a building 200 feet tall, with an initial velocity of 100 feet/second, is given by the equation $h=-16t^2+100t+200$, where h represents the height of the object after t seconds. At what time(s) will the object be 300 feet above the ground? (Round answer to nearest hundredth.)

9. Jack drove 600 miles to a convention in Washington D. C. On the return trip he was able to increase his speed by 10 mph and save 3 hours of driving time. (a) Find his rate for each direction. (b) Find his time for each direction.

10. When tickets for a rock concert cost $12, the average attendance was 500 people. Projections showed that for each $1 increase in ticket prices, 50 less people would attend. At what ticket price would the receipts be $5600.

1.4 Homework Answers: 1. $\{2\pm\sqrt{10}\}$ 2. {0, 3} 3. base: 22 in; height: 5 in 4. 78 ft by 36 ft 5. 16 teams 6. (a) $x^2+\frac{17}{4}x-\frac{15}{4}=0$; (b) $4x^2+17x-15=0$ (answers vary) 7. 8 seconds 8. 1.25 and 5 seconds 9. (a) 40 mph; 50 mph (b) 15 hours; 12 hours 10. $14 ($2 increase)

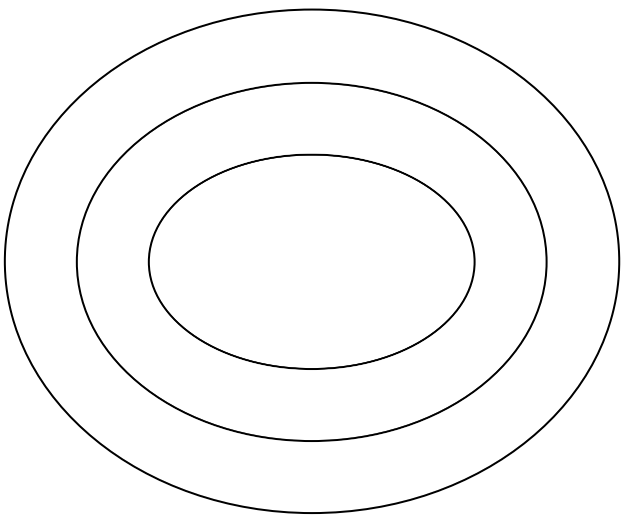
1.5 Complex Numbers

In this section you will learn to:

- add, subtract, multiply and divide complex numbers
- simplify complex numbers
- find powers of *i*
- solve quadratic equations with complex roots

The **imaginary unit *i*** is defined as $i = \sqrt{-1}$, where $i^2 = -1$.

The set of all numbers in the form **a + *bi*** with real numbers *a* and *b* and *i*, the imaginary unit, is called the set of **complex numbers**. (**Standard form** for a complex number is ***a* + *bi***.)

THE COMPLEX NUMBER SYSTEM		
Real Numbers		**Imaginary Numbers**
Rational Numbers 	**Irrational Numbers**	

Example 1: Simplify each imaginary number below:

$\sqrt{-9}$ = _____ $\sqrt{-36}$ = _____ $\sqrt{-8}$ = _____ $\sqrt{-27}$ = ______ $\sqrt{\frac{-12}{25}}$ = _____

Equality of Complex Numbers: $a + bi = c + di$ if and only if $a = c$ and $b = d$.

Addition of Complex Numbers: $(a+bi)+(c+di) = (a+c)+(b+d)i$

Subtraction of Complex Numbers: $(a+bi)-(c+di) = (a+c)-(b+d)i$

Multiplication of Complex Numbers: $(a+bi)(c+di) = (ac-bd)+(ad+bc)i$

NOTE: Add, subtract, or multiply complex numbers as if they were binomials.

Example 2: Add: $(2 - 3i) + (-4 + 5i)$

Example 3: Subtract: $(2 - 3i) - (-4 + 5i)$

Example 4: Multiply: $(2 - 3i)(-4 + 5i)$

Recall: $(a + b)$ and $(a - b)$ are c**onjugates.** Therefore, $(a + b)(a - b) = a^2 - b^2$.

$a + bi$ and $a - bi$ are called **complex conjugates** therefore:

$(a + bi)(a - bi) =$

Example 5 (Division): Simplify $\dfrac{2-3i}{-4+5i}$ (Hint: Multiply numerator and denominator by **conjugate** of the denominator.)

Example 6 (Division): Find the reciprocal of $4 + 3i$.

Example 7: Simplify each of the following powers of i.

$i^3 =$ _____ $i^4 =$ _____ $i^{13} =$ _____ $i^{102} =$ _____ $i^{-1} =$ _____ $i^{-6} =$ _____

Recall: $\sqrt{-1} = i$. Therefore $\sqrt{-b} = \sqrt{(-1)b} = \sqrt{-1}\sqrt{b} = i\sqrt{b}$.

Example 8: Simplify and write each answer in **standard form.**

(a) $\sqrt{-27} + \sqrt{-18}$

(b) $\sqrt{-4} \cdot \sqrt{-9}$

(c) $(-2 + \sqrt{-5})^2$

(d) $(\sqrt{-1} + \sqrt{-3})(\sqrt{-1} - \sqrt{-3})$

(e) $\dfrac{-12 + \sqrt{-12}}{12}$

Example 8: Solve: $a^2 + 4a + 8 = 0$

Example 9: Solve: $5x^2 + x = -5$

1.5 Homework Problems

1. Simplify each expression: (a) $\sqrt{-169}$ (b) $\sqrt{-200}$ (c) $\sqrt{-108}$ (d) $\sqrt{-\frac{50}{49}}$

   (e) $6\sqrt{-\frac{27}{4}}$ (f) i^5 (g) i^{26} (h) i^{-7}

2. Perform the indicated operation for each problem below. Simplify each problem and write the answer in standard form.

   (a) $(3-7i)+(-8-4i)$ (b) $(6-22i)-(-9+5i)$ (c) $(3-4i)(-8+5i)$

   (d) $(4-5i)^2$ (e) $\frac{5}{i}$ (f) $\frac{-i}{1+i}$ (g) $\frac{2-i}{3+i}$ (h) $(2+i)^3$

   (i) $\sqrt{-200}+\sqrt{-8}$ (j) $\sqrt{-16}\cdot\sqrt{-25}$ (k) $\sqrt{-3}\cdot\sqrt{-27}$

   (l) $-(2-5i)^2$ (m) $\frac{1}{(1-\sqrt{-4})^2}$ (n) $\frac{-18-\sqrt{-27}}{9}$

3. Simplify: $3(2+3i)^2-\frac{5}{1+2i}$

4. Simplify: $(x-2+3i)(x-2-3i)$

5. Find the reciprocal of 7 + 3i.

6. Solve each of the following:

   (a) $x^2=2x-17$ (b) $2x^2+5=6x$ (c) $9x^2-24x+8=0$ (d) $x^2+11x=-49$

1.5 Homework Answers: 1. (a) 13i (b) $10i\sqrt{2}$ (c) $6i\sqrt{3}$ (d) $\frac{5}{7}i\sqrt{2}$ (e) $9i\sqrt{3}$ (f) i (g) -1 (h) i 2. (a) -5 – 11i (b) 15 – 27i (c) -4 + 47i (d) -9 – 40i (e) -5i (f) $-\frac{1}{2}-\frac{1}{2}i$ (g) $\frac{1}{2}-\frac{1}{2}i$ (h) 2 + 11i (i) $12i\sqrt{2}$ (j) -20 (k) -9 (l) 21 + 20i (m) $-\frac{3}{25}+\frac{4}{25}i$ (n) $-2-\frac{\sqrt{3}}{3}i$ 3. -16 + 38i 4. $x^2-4x+13$ 5. $\frac{7}{58}-\frac{3}{58}i$ 6. (a) $1\pm 4i$; (b) $\frac{3}{2}\pm\frac{1}{2}i$; (c) $\frac{4\pm 2\sqrt{2}}{3}$ or $\frac{4}{3}\pm\frac{2\sqrt{2}}{3}$;

(d) $-\frac{11}{2}\pm\frac{5\sqrt{3}}{2}i$

1.6 Polynomial & Radical Equations

In this section you will learn to:

- solve polynomial equations using factoring
- solve radical equations
- solve equations with rational exponents
- solve equations in quadratic form

A **polynomial equation** in x: $p(x) = q(x)$, where $p(x)$ and $q(x)$ are polynomials.

The **general form of a polynomial is** $p(x) = 0$

The **degree of a polynomial** is the highest degree of any term in the polynomial.

Solving Polynomials by Factoring:

Example 1: Solve: $4x^4 = 12x^2$

Example 2: Solve: $9y^3 + 8 = 4y + 18y^2$

Example 3: Solve: $x^3 - 2x^2 - 6x = 0$

Solving Radical Equations:

Example 4: Solve: $x - \sqrt{x+11} = 1$

Steps:

1. Isolate radical(s).
2. Square both sides.
3. Expand.
4. Solve for x.
5. **Check answer!**

Example 5: Solve: $\sqrt{x+5}-\sqrt{x-3}=2$ (Hint: Move one of the radicals to the other side and repeat Example 4 Steps 1-3 after expanding.)

<u>**Solving Equations in Quadratic Form (using substitution)**</u>**:**

Example 6: Solve: $x-8\sqrt{x}-20=0$

Example 7: Solve: $4x^4=13x^2-9$

Example 8: Solve: $6x^{\frac{2}{5}}+11x^{\frac{1}{5}}+3=0$

Example 9: Solve: $x^{-2}-7x^{-1}-8=0$

Example 10: Try this alternative shortcut to substitution using this example for the following problems:

$$x^2 - 3x - 10 = 0 \Rightarrow (x-5)(x+2) = 0 \Rightarrow x = 5 \quad or \quad x = -2$$

$x^4 - 3x^2 - 10 = 0$

$x - 3\sqrt{x} - 10 = 0$

$x^{\frac{2}{3}} - 3x^{\frac{1}{3}} - 10 = 0$

$x^{-2} - 3x^{-1} - 10 = 0$

$(x+2)^2 - 3(x+2) - 10 = 0$

<u>Solving Equations with Rational Exponents</u>: Recall: $\sqrt{x^2} = |x| = \begin{cases} x \text{ if } x \ge 0 \\ -x \text{ if } x < 0 \end{cases}$, and

$\sqrt{x} = x^{\frac{1}{2}}$, $\sqrt[3]{x} = x^{\frac{1}{3}}$, $\sqrt[3]{x^2} = x^{\frac{2}{3}}$, ... $\sqrt[m]{x^n} = x^{\frac{n}{m}}$

Example 11: Solve: $8x^{\frac{5}{3}} - 24 = 0$

Example 12: Solve: $(x+5)^{\frac{2}{3}} = 4$

1.6 Homework Problems

Solve each of the equations below using any appropriate method:

1. $5x^5 - 25x^3 = 0$
2. $x^3 + 6x = 5x^2$
3. $\frac{12}{x} - \frac{x}{2} = x - 3$
4. $x^4 - 2x^2 + 1 = 0$
5. $\frac{3x}{2} - \frac{2x}{x-1} + 3 = x$
6. $\sqrt{2x-3} + 1 = x$
7. $2x - 5\sqrt{x} = -3$
8. $x^{\frac{2}{3}} + 5x^{\frac{1}{3}} - 6 = 0$
9. $\sqrt{x+5} + \sqrt{x} - 1 = 0$
10. $\sqrt{5-x} + \sqrt{5+x} = 4$
11. $\sqrt{2x+3} = 1 - \sqrt{x+1}$
12. $2x^4 = 50x^2$
13. $2x^3 + 9 - 18x = x^2$
14. $3x^{\frac{3}{4}} - 24 = 0$
15. $(x-7)^{\frac{2}{3}} = 4$
16. $x^3 + 3x^2 = 2x + 6$
17. $\sqrt{x^2 + 1} = \frac{\sqrt{-7x+11}}{\sqrt{6}}$

Solve each of the following equations using substitution:

18. $x^{-2} - x^{-1} = 20$
19. $2x - 5\sqrt{x} = -3$
20. $x^3 - 7x^{\frac{3}{2}} - 8 = 0$

1.6 Homework Answers: 1. $\{0, \pm\sqrt{5}\}$ 2. {0, 2, 3} 3. {-2, 4} 4. $\{\pm 1\}$ 5. {2, -3} 6. {2} 7. $\left\{\frac{9}{4}, 1\right\}$ 8. {-216, 1} 9. ϕ 10. $\{\pm 4\}$ 11. {-1} 12. {-5, 0, 5} 13. $\left\{-3, \frac{1}{2}, 3\right\}$ 14. {16} 15. {-1, 15} 16. $\{-3, \pm\sqrt{2}\}$ 17. $\left\{-\frac{5}{3}, \frac{1}{2}\right\}$ 18. $\left\{-\frac{1}{4}, \frac{1}{5}\right\}$ 19. $\left\{\frac{9}{4}, 1\right\}$ 20. {4}

1.7 Inequalities

In this section you will learn to:

- use interval notation
- understand properties of inequality
- solve linear (and compound) inequalities
- solve polynomial inequalities
- solve rational inequalities

<u>Interval Notation</u>:

Inequality	Graph	Interval Notation	Set Builder Notation
$x < 4$			
$x \geq -3$			
$-3 < x \leq 2$			
$x \leq -3$ or $x > 5$			
all real #'s			

Intersection	Union

<table>
<tr><th colspan="3">Properties of Inequalities</th></tr>
<tr><th>+/- Property of Inequality</th><th colspan="2">Multiplication/Division Property of Inequality</th></tr>
<tr><td rowspan="2">If $a < b$, then

$a + c < b + c$

$a - c < b - c$

(Adding or subtracting does not affect the > or < sign.)</td><td>$c > 0$ (c is positive)</td><td>$c < 0$ (c is negative)</td></tr>
<tr><td>If $a < b$, then

$ac < bc$

$\frac{a}{c} < \frac{b}{c}$
(Multiplying or dividing by a positive number does not affect the > or < sign.)</td><td>If $a < b$, then
$ac > bc$

$\frac{a}{c} > \frac{b}{c}$

(When multiplying or dividing by a negative number, reverse the > or < sign.)</td></tr>
</table>

Example 1: Solve and graph the inequality below. Write the answer using interval notation.

$2 - 3x \le 5$

Example 2: Solve and graph the inequality below. Write the answer using interval notation.

$5 - (7x + 19) > 13(x - 5)$

Example 3: Solve the compound ("and") inequality $-3 < \frac{2}{3}x + 1 \le 5$. Write answer using interval notation.

(a) Solve by isolating the variable *x*.

(b) Solve by writing each inequality separately.

Example 4: Avis charges \$40/day plus \$.10/mile to rent a car. Hertz charges \$50/day plus \$.08/mile. When is Avis a better deal if you are renting a car for three days?

A **polynomial inequality** is any inequality of the form:

$f(x) < 0$ (graph is below the x-axis)

$f(x) \le 0$ (graph is on or below the x-axis)

$f(x) > 0$ (graph is above the x-axis)

$f(x) \ge 0$ (graph is on or above the x-axis)

where f is a **polynomial function**.

Example 5: Solve $x^2 - 6 > 5x$. Write the solution using interval notation.

<u>Steps for Solving Polynomial Inequalities</u>:

1. Express as $f(x) > 0$ or $f(x) < 0$.
   (Get 0 on right side.)

2. Set $f(x) = 0$ and solve for x to get
   Boundary Points.

3. Plot the boundary points on a number
   line to obtain **Intervals.**

4. **Test Values** within each interval and
   evaluate $f(x)$ for each value.
   If $f(x) > 0$, then $f(x)$ is + for interval.
   If $f(x) < 0$, then $f(x)$ is – for interval.

5. Write the solution using interval notation.
   Check the solution on your calculator.

Example 6: Solve $x^3 + x^2 - 9 \le 9x$. Write the solution using interval notation.

Example 7: Solve $x^3 + 2x^2 \ge 4x + 8$. Write the solution using interval notation.

A **rational inequality** is any inequality of the form:

$f(x) < 0$ (graph is below the *x*-axis)

$f(x) \le 0$ (graph is on or below the *x*-axis)

$f(x) > 0$ (graph is above the *x*-axis)

$f(x) \ge 0$ (graph is on or above the *x*-axis)

where *f* is a **rational function**. ($f(x) = \dfrac{p(x)}{q(x)}$, where *p* and *q* are polynomials and $q(x) \ne 0$)

Example 8: Solve $\dfrac{x-2}{x+5} \ge 0$. Write the solution using interval notation.

<u>Steps for Solving Rational Inequalities</u>:

1. Express as $f(x) > 0$ or $f(x) < 0$.
   (Get 0 on right side.)

*2. Find values that make the numerator & the denominator = 0. These are the **Boundary Points. (Note Restrictions!)**

3. Plot the boundary points on a number line to obtain **Intervals.**

4. **Test Value** within each interval and evaluate $f(x)$ for each value.
   If $f(x) > 0$, then $f(x)$ is + for interval.
   If $f(x) < 0$, then $f(x)$ is – for interval.

5. Write the solution using interval notation.
   Check the solution on your calculator.

Example 9: Solve $\frac{x}{x+2} \geq 2$. Write the solution using interval notation.

Example 10: A ball is thrown vertically from a rooftop 240 feet high with an initial velocity of 64 feet per second. During which time period will the ball's height exceed that of the rooftop? (Use $h(t) = -16t^2 + v_0 t + s_0$ where v_0 = initial velocity, s_0 = initial height/position, and t = time. You may also want to graph this function on your calculator using the viewing rectangle [0, 10, 1] by [-100, 500, 100]).

1.7 Homework Problems

Solve each of the inequalities below and write the answer using interval notation.

1. $3+5x \le 2(1+3x)$
2. $3(x+2) \le 4(x+5)$
3. $\dfrac{3(x+3)}{2} < \dfrac{2(x+7)}{3}$
4. $\dfrac{2}{3}x - x \le -\dfrac{3}{2}(x-5)$
5. $\dfrac{1}{4}x + \dfrac{2}{3}x - x > \dfrac{1}{2} + \dfrac{1}{2}(x+1)$
6. $2+x < 3x-2 \le 5x+2$
7. $0 \le \dfrac{3+x}{2} < 4$
8. $x^2 - 2x - 8 > 0$
9. $2x^2 + x - 3 \le 0$
10. $x^3 + x^2 \le 4x+4$
11. $x^3 \ge 9x^2$
12. $x^3 - 2x^2 - 4x + 8 \le 0$
13. $9x^2 - 6x + 1 < 0$
14. $\dfrac{2}{x} < 4$
15. $\dfrac{x-4}{x+3} > 0$
16. $\dfrac{4-2x}{3x+4} \le 0$
17. $\dfrac{-x+2}{x-4} \ge 0$
18. $\dfrac{x+1}{x+3} \le 2$
19. $\dfrac{3}{x-2} \ge 5$
20. $\dfrac{x}{2x-1} - 1 \ge 0$

1.7 Homework Answers: 1. $[1, \infty)$ 2. $[-14, \infty)$ 3. $\left(-\infty, \dfrac{1}{5}\right)$ 4. $\left(-\infty, \dfrac{45}{7}\right]$ 5. $\left(-\infty, -\dfrac{12}{7}\right)$ 6. $(2, \infty)$ 7. $[-3, 5)$ 8. $(-\infty, -2) \cup (4, \infty)$ 9. $\left[-\dfrac{3}{2}, 1\right]$ 10. $(-\infty, -2] \cup [-1, 2]$ 11. $\{0\} \cup [9, \infty)$ 12. $(-\infty, -2] \cup \{2\}$ 13. ϕ 14. $(-\infty, 0) \cup \left(\dfrac{1}{2}, \infty\right)$ 15. $(-\infty, -3) \cup (4, \infty)$ 16. $\left(-\infty, -\dfrac{4}{3}\right) \cup [2, \infty)$ 17. $[2,4)$ 18. $(-\infty, -5] \cup (-3, \infty)$ 19. $\left(2, \dfrac{13}{5}\right]$ 20. $\left(\dfrac{1}{2}, 1\right]$

1.8 Absolute Value

In this section you will learn to:

- solve absolute value equations
- solve absolute value inequalities
- apply absolute inequalities

The **absolute value** of a real number x is denoted by $|x|$, and is defined as follows:

If $x \geq 0$ (non-negative number), then $|x| = x$.

If $x < 0$ (negative number), then $|x| = -x$.

Absolute Value Equations: If $k \geq 0$ and $|x| = k$, then $x = k$ or $x = -k$.

Solving Absolute Value Equations:

Example 1: Solve: $|2x+3| = 7$

Example 2: Solve: $-2\left|3-\frac{x}{5}\right|-1=-3$

Example 3: Solve: $|x+1|+6=2$

Example 4: Solve: $16 = 7|3x+14|+2$

Example 5: Solve: $|x^2-3x-11|-3=4$

Example 6: Solve (using guess and check) and graph each of the following:

$|x| = 2$ $\qquad$ $|x| > 2$ $\qquad$ $|x| < 2$

Which solutions involve "and"? Which use the concept of "or"?

Absolute Value Inequalities: If $k > 0$, then $|x| \geq k$ is equivalent to $x \geq k$ **or** $x \leq -k$ (union).
If $k > 0$, then $|x| \leq k$ is equivalent to $x \leq k$ **and** $x \geq -k$ (intersection).

<u>Solving Absolute Value Inequalities</u>:

Example 7: Solve and graph the inequality below. Write answer in **interval notation.**

$$|5x - 4| < 3$$

Example 8: Solve and graph the inequality below. Write answer in **interval notation.**

$$5 \leq \left|\frac{x}{2} + 3\right|$$

Example 9: Solve: $-3|x - 1| + 2 \leq 8$

Example 10: Solve: $0 < |x + 2| \leq 5$

Example 11: The temperatures on a summer day satisfy the inequality $|t - 74°| \le 10°$, where t is the temperature in degrees Fahrenheit. Express this range without using absolute value symbols.

Example 12: A weight attached to a spring hangs at rest a distance of x inches off of the ground. If the weight is pulled down (stretched) a distance of L inches and released, the weight begins to bounce and its distance d off of the ground at any time satisfies the inequality $|d - x| \le L$. If x equals 4 inches and the spring is stretched 3 inches and released, solve the inequality to find the range of distances from the ground the weight will oscillate.

1.8 Homework Problems:

Solve each of the absolute value equations below:

1. $|x-5|+8=12$

2. $-2|x+4|+3=-1$

3. $\left|\frac{2}{3}x+\frac{5}{6}\right|-\frac{7}{12}=\frac{11}{12}$

4. $-5|2m-7|+2=-13$

5. $|x^2-2x-25|=10$

Solve each inequality. Write the solution using interval notation.

6. $3|x+4|+5<8$

7. $-3|x-5|>-12$

8. $\frac{|3x+2|}{-4}\le -1$

9. $-3\le -2\left|3-\frac{x}{5}\right|-1$

10. $|4-3x|+12<7$

11. $-1>\frac{|2x-3|}{-3}$

12. A Steinway piano should be placed in room where the relative humidity h is between 38% and 72%. Express this range with an inequality containing an absolute value.

13. The optimal depths d (in feet) for catching a certain type of fish satisfy the inequality $28|d-350|-1400<0$. Find the range of depths that offer the best fishing.

1.8 Homework Answers: 1. {1, 9} 2. {-6, -2} 3. {-3.5, 1} 4. {2, 5} 5. {-5, -3, 5, 7} 6. (-5, -3)

7. (1, 9) 8. $(-\infty,-2]\cup\left(\frac{2}{3},\infty\right)$ 9. [10, 20] 10. ϕ 11. $(-\infty,0)\cup(3,\infty)$ 12. $|h-55|<17$

13. (300,400)