2.5 Marginal Cost and Revenue

* Marginal Analysis

The cost function, \( C(q) \), gives the total cost of producing a quantity \( q \) of some good. Define

\[
\text{Marginal Cost} = MC(q) = C'(q),
\]

which gives us that

\[
\text{Marginal Cost} \approx C(q + 1) - C(q).
\]

The revenue function, \( R(q) \), gives the total revenue received from a firm from selling a quantity, \( q \), of some good. Define

\[
\text{Marginal Revenue} = MR(q) = R'(q),
\]

which gives us that

\[
\text{Marginal Revenue} \approx R(q + 1) - R(q).
\]

**Example 1** In the figure below, is marginal cost greater at \( q = 5 \) or at \( q = 30 \)? At \( q = 20 \) or at \( q = 40 \)?
Example 2 In the figure below, estimate the marginal revenue when the level of production is 750 units and interpret it.

\[ R(q) \]

Example 3 For \( q \) units of a product, a manufacturer’s cost is \( C(q) \) dollars and revenue is \( R(q) \) dollars, with \( C(500) = 7200 \), \( R(500) = 9400 \), \( MC(500) = 15 \), and \( MR(500) = 20 \).

(a) What is the profit or loss at \( q = 500 \)?

(b) If production is increased from 500 to 501 units, by approximately how much does profit change?

Example 4 Let \( C(q) \) represent the total cost of producing \( q \) items. Suppose \( C(1000) = 500 \) and \( C'(1000) = 25 \). Estimate the total cost of producing 1001 items, 999 items and 1100 items.
Example 5 Let $C(q)$ represent the cost and $R(q)$ represent the revenue, in dollars, of producing $q$ items.

(a) If $C'(50) = 24$ and $R'(50) = 35$, approximately how much profit is earned by the 51st item?

(b) If $C'(100) = 38$ and $R'(100) = 35$, should the company produce the 101st item? Why or why not?

Example 6 A company’s cost of producing $q$ liters of a chemical is $C(q)$ dollars; this quantity can be sold for $R(q)$ dollars. Suppose $C(2000) = 5930$ and $R(2000) = 7780$.

(a) What is the profit at a production level of 2000?

(b) If $MC(2000) = 2.1$ and $MR(2000) = 2.5$, what is the approximate change in profit if $q$ is increased from 2000 to 2001? Should the company increase or decrease production from $q = 2000$?

(c) If $MC(2000) = 4.77$ and $MR(2000) = 4.32$, should the company increase or decrease production from $q = 2000$?

Example 7 The cost and revenue functions of a bicycle manufacture are shown in the following graph.

(a) For how many bicycle does the manufacture make a profit?

(b) Does it cost more to produce the 101st bicycle or 301st bicycle? Why?

(c) Should the manufacture increase production from 200 to 201? Why?