1.9 Proportional and Power Functions

*** Proportionality**

We say y is (directly) **proportional** to x if there is a nonzero constant k such that

$$y = kx.$$

This k is called the constant of proportionality.
We say y is **inversely proportional** to x if there is a nonzero constant k such that

$$y = \frac{k}{x}.$$

Or equivalently, if the product of x and y equals a constant k, then y is **inversely proportional** to x.

Example 1 *The blood mass of a mammal is proportional to its body mass.*

(a) Write a formula for blood mass, B, as a function of body mass, M.

(b) A rhinoceros with body mass 3000 kilograms has blood mass of 150 kilograms. Use this information to find the constant of proportionality.

(c) Estimate the blood mass of a human with body mass 70 kilograms.

Example 2 *The number of animal species of a certain body length, N, is inversely proportional to the square of the body length, L. Write a formula for N as a function of L. Are there more species at large lengths or at small lengths?*

Example 3 *Use the following tables to determine whether $f(x)$ and $g(x)$ are proportional or inversely proportional to x? If so, find the constant of proportionality and write a formula for the corresponding function.*

(a)

x	-3	0	6	9	12
$f(x)$	60	0	-120	-180	-240

(b)

x	-2	2	6	10	14
$g(x)$	105	-105	-35	-21	-15

*** Power Functions**

We say $Q(x)$ is a **power function** of x if $Q(x)$ is proportional to a constant power of x. If k is the constant of proportionality, and if p is the power, then

$$Q(x) = k \cdot x^p.$$

Example 4 *Which of the following are power functions? For those which are, write the function in the form $y = kx^p$, and give the coefficient k and the exponent p.*

(a) $y = \dfrac{10}{x^4}$

(b) $y = 6 \cdot 4^x$

(c) $y = 9\sqrt{x}$

(d) $y = (2x^3)^2$

(e) $y = x^8 + 1$

(f) $y = \dfrac{5}{3\sqrt{x}}$

(g) $y = \dfrac{x}{9}$

*** Graphs of Power Functions**

*** Quadratic Functions and Polynomials**

Sums of power functions with nonnegative integer exponents are called **polynomials**, which are functions of the form

$$y = p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0.$$

Here, n is a nonnegative integer, called the **degree** of the polynomial, and a_n is a nonzero number called the **leading coefficient**. We call $a_n x^n$ the **leading term**.
If $n = 2$, the polynomial is called **quadratic**.

Example 5 *Which of the following functions are polynomial functions? For those which are, give the degree n and the leading coefficient a_n.*

(a) $3x^{-2} + 1$

(b) $7x^{10} + x^2$

(c) $2^x + 3$

(d) $2\sqrt{x} + x - 1$

(e) $8x^6 + x^2 - 4x + 2 - 8x^6$