1.5 Exponential Functions

*** The General Exponential Function**

We say that P is an **exponential function** of t with base a if

$$P = P_0 a^t,$$

where P_0 is the initial quantity (when $t = 0$) and a is the factor by which P changes when t increases by 1. If $a > 1$, we have **exponential growth**; if $0 < a < 1$, we have **exponential decay**. The factor a is given by

$$a = 1 + r,$$

where r is the decimal representation of the percent rate of change; r may be positive (for growth) or negative (for decay).

Example 1 *World population is approximated by $P = 6.4(1.0126)^t$, with P in billions and t in years since 2004.*

(a) What is the yearly percent rate of growth of the world population?

(b) What was the world population in 2004? What does this model predict for the world population in 2012?

*** Comparison Between Linear and Exponential Functions**

A **linear** function has a constant rate of change.
An **exponential** function has a constant percent, or relative, rate of change.

Example 2 *The annual net sales for a chocolate company in 2008 was 5.1 billion dollars. In each of the following cases, write a formula for the annual net sales, S, of this company as a function of t, where t represents the number of years after 2008.*

(a) The annual net sales increases by 1.2 billion dollars per year.

(b) The annual net sales decreases by 0.4 billion dollars per year.

(c) The annual net sales increases by 4.3% *per year.*

(d) The annual net sales decreases by 1% *per year.*

*** Recognizing Data from an Exponential Function**

> Values of t and P in a table could come from an exponential function $P = P_0 a^t$ if ratios of P values are constant for equally spaced t values.

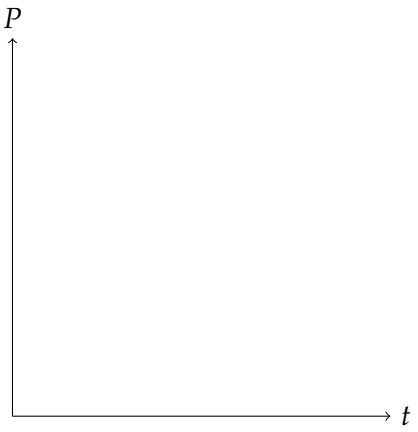
Example 3 *Which of the following functions in the following table could be linear, exponential, or neither? Find formulas for those functions.*

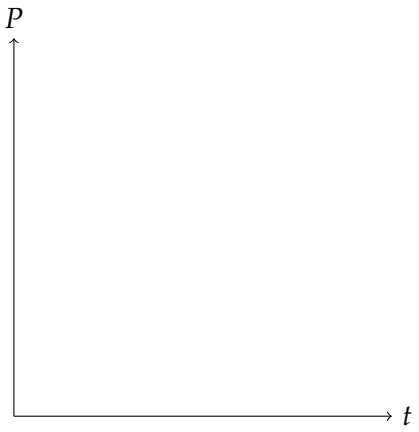
x	-2	-1	0	1	2
$f(x)$	500	600	700	800	900
$g(x)$	14	20	24	29	35
$h(x)$	16	24	36	54	81

*** The Families of Exponential Functions and Number e**

The formula $P = P_0 a^t$ gives a family of exponential functions with parameters P_0 (the initial quantity) and a (the base).
If $a > 1$, then the function is increasing.
If $0 < a < 1$, then the functions is decreasing.
The larger a is, the faster the function grows; the closer a is to 0, the faster the functions decays.

The most commonly used base is the number $e = 2.71828...$, which is called the natural base.





Example 4 *Give a possible formula for the function which is represented by the following graph.*