Homework for Math 152H-1 September 20

Reading: A function $f(x)$ is called continuously differentiable if $f'(x)$ is continuous. The function $y = x^2 sin\frac{1}{x}$ is an example of a differentiable function, which is not continuously differentiable. The relationship between continuity and differentiability is thus, $f'(x)$ exists implies $f(x)$ is continuous, but $f'(x)$ can be either continuous or not continuous. If $f''(x)$ (the derivative of the derivative) exists, then both $f'(x)$ and $f(x)$ are continuous, but $f''(x)$ may or may not be.

We didn't get to this problem in class. So here's the solution:

(6) For what value of c is

$$y = \begin{cases} sin\, x & x \leq 0 \\ c(x^2 + x) & x \geq 0 \end{cases}$$

differentiable at 0? (**Hint:** can you use your previous results?)

First note that since $sin\, 0 = 0$ and $c(0^2 + 0) = 0$, the function is continuous. Away from 0 we can take the derivatives to get:

$$y' = \begin{cases} cos\, x & x < 0 \\ c(2x + 1) & x > 0 \end{cases}$$

In fact, both $sin\, x$ and $c(x^2 + x)$ are defined on the whole real line. So both prescribe a slope for the tangent line at 0 and this can be found from the derivatives above. (This differs from $x^2 sin\, \frac{1}{x}$, but as these functions don't have any problems at 0, we can work out what happens at 0 from what happens nearby). To have a derivative, the slopes must exist and be equal, so set $cos\, 0 = c(2 \cdot 0 + 1)$ or $1 = c$. If you choose a different value of c, you will have a continuous function with two different slopes at 0, i.e. its graph will have a kink.

Homework: The homework today involves computations of derivatives. The relevant sections in the book are 3.2, 3.4, and the beginning of 3.5 (this sounds like a lot, but mainly the sections consist of example after example). If you have trouble you should check there. Note:

$$\frac{d^2}{dx^2}\, f(x) = \frac{d}{dx}\left(\frac{d}{dx}\, f(x)\right)$$

(1) $\frac{d}{dx}\,\left(3\,x^2 + 2\sqrt{x} - 3\right)$

(2) $\frac{d^2}{dx^2}\,\left(\frac{12}{x^3} - sin\, x\right)$

(3) $\frac{d}{dx}\,\left(2\,x^\pi - \frac{2}{x + x^2}\right)$

(4) $\frac{d}{dx}\,\left(\left(1 + \sqrt{x}\right)(cos\, x + sin\, x)\right)$

(5) $\frac{d}{dx}\,\left(tan\, x + \frac{x^2 + 1}{2x + 1}\right)$

(6) $\frac{d^2}{dx^2}\, 3(sin\, 2x)(csc\, 2x)$

(7) $\frac{d}{dx}\,\left((x + \frac{1}{x}) \cdot sec\, x\right)$

(8) $\frac{d}{dx}\,\left(\sqrt{1 + 3x} - sin\, 2x\right)$

(9) $\frac{d}{dx}\, cos(\sqrt{x})$

(10) $\frac{d}{dx}(x + cos\, x)^{101}$

(11) $\frac{d}{dx}\,\left(x^2\, sin\, x + 2x\, cos\, x - 2\, sin\, x\right)$

(12) $\frac{d}{dx}\,\frac{tan\, 3x}{1 + tan\, 2x}$

(13) $\frac{d}{dx}\sqrt{\frac{sin\, x}{2 + cos^2 x}}$

(14) $\frac{d^{101}}{dx^{101}}\, cos\, x$

(15) $\frac{d}{dx}\ sin(\sqrt{1+cos\,x})$

(16) $\frac{d}{dx}\ \left(1+sin^2(\frac{1}{2x+1})\right)^{12}$

(17) If we take the derivative of $y = 3x+2$ we get $y' = 3$. If we take the derivative of this we get $y'' = 0$. In fact, If $\frac{d^n}{dx^n}P(x) = 0$, then $P(x)$ is a polynomial (as we will see later). Let $P(x) = a_m x^m + a_{m-1}x^{m-1} + \ldots a_1\,x + a_0$ with $a_m \neq 0$. How many derivatives must we take before we get 0?

(18) Prove the product formula:

$$\frac{d}{dx}\big(f(x)\cdot g(x)\big) = \frac{d}{dx}\big(f(x)\big)\cdot g(x) + f(x)\cdot\frac{d}{dx}\big(g(x)\big)$$

by completing the following computation:

$$\lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x)}{h} = \lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x+h)+f(x)g(x+h)-f(x)g(x)}{h} =?$$