Homework for Math 152H-1 September 20

Reading: Read section 3.1 pgs 152-154, sec 3.3

Homework:

Some useful identities:

$$x^2 - a^2 = (x-a)(x+a)$$
$$x^n - a^n = (x-a)(x^{n-1} + a\,x^{n-2} + a^2\,x^{n-3} + \cdots + a^{n-2}\,x + a^{n-1})$$
$$sin(a+h) = sin(a)\,cos(h) + cos(a)\,sin(h)$$
$$cos(a+h) = cos(a)\,cos(h) - sin(a)\,sin(h)$$
$$cos(x) = 1 - 2\,sin^2\left(\tfrac{x}{2}\right)$$

(1) Use the definition of the derivative and the identities above to calculate the derivative *functions* for

$$y = 3x+1 \quad y = 3 \quad y = x^2 + x$$
$$y = \sqrt{x} \quad y = \tfrac{1}{x^2} \quad y = x^n$$

The derivative function tells you the slope of the tangent line at each point. Find the *equation* of the tangent line to each of the above functions at $x = 1$.

(2) Use the definition to calculate the derivative of $y = sin\,x$. You should use the formula:

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

and one of the above identities. At some point you will also need the last identity for $cos\,x$. Once you have done this verify that the derivative of $cos\,x$ is $-sin\,x$ (it's the same procedure, but with different identites).

(3) Show that $y = |x|$ is not differentiable at 0 (show that the necessary limit does not exist by looking at right hand and left hand limits!).

(4) Show that the following function is not differentiable at 0:

$$h(x) = \begin{cases} 1 & x = 0 \\ -x^2 & x \neq 0 \end{cases}$$

Can you do this without using the definition?

(5) Is the following function differentiable at 0:

$$y = \begin{cases} x^2 sin\,\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

How about

$$y = \begin{cases} x\,sin\,\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

(6) For what value of c is

$$y = \begin{cases} sin\,x & x \leq 0 \\ c(x^2 + x) & x \geq 0 \end{cases}$$

differentiable at 0? (**Hint:** can you use your previous results?)