**Homework for Math 152H-1 September 11**

**Homework:** Recall the limit we mentioned at the end of class:

if $f(x) \to 0$ as $x \to a$ then $\lim_{x\to a} \dfrac{sin\, f(x)}{f(x)} = 1.$

The denominator *must* be the same as what you are taking the sine of! Also recall the following identities:

$$tan\, x = \frac{sin\, x}{cos\, x} \qquad sec\, x = \frac{1}{cos\, x} \qquad csc\, x = \frac{1}{sin\, x} \qquad cot\, x = \frac{cos\, x}{sin\, x}$$

The following example will be helpful:

$$\lim_{x\to 0} \frac{sin\, 2x}{x} = \lim_{x\to 0} \frac{sin\, 2x}{2x} \cdot 2 = 1 \cdot 2$$

Compute

(1) $\lim_{x\to 0} \dfrac{sin\, \sqrt{3}x}{\sqrt{3}x}$

(2) $\lim_{x\to 0} \dfrac{sin\, 3x}{5x}$

(3) $\lim_{x\to 0} \dfrac{tan\, \sqrt{3}x}{x}$

(4) $\lim_{x\to 0} 2x^2 cot\, x\, csc\, x$

(5) $\lim_{x\to 0} \dfrac{2x + sin\, x^2}{x}$

(6) $\lim_{x\to 1} \dfrac{sin(x^2 - x - 2)}{x - 1}$

(7) $\lim_{x\to 0} \dfrac{sin(2\, sin\, x)}{x}$

(8) $\lim_{x\to 0} \dfrac{sin\, 5x}{tan\, 4x}$

(9) $\lim_{x\to 2} \dfrac{sin(\sqrt{x+2} - 2)}{x - 2}$

(10) $\lim_{x\to\infty} \sqrt{x} sin(x^{-\frac{1}{2}})$

(11) $\lim_{x\to 1^-} \dfrac{sin(\sqrt{(x-1)^2})}{x - 1}$

(12) Use the following identity $cos(\theta) = 1 - 2\, sin^2(\frac{\theta}{2})$ to compute:

$$\lim_{x\to 0} \frac{1 - cos\, x}{x} \qquad \lim_{x\to 0} \frac{1 - cos\, x}{x^2}$$

( 13) Show that the following limit does not exist:

$$\lim_{x\to 0^+} sin\, \frac{1}{x}$$

**Hint:** Find two sequences converging to zero, such that $sin\, \frac{1}{x}$ converges to two different numbers. To do this note that as $x \to 0^+$, $\frac{1}{x} \to \infty$, but how does $sin\, y$ behave as $y \to \infty$? This is an example of a function where even the right and left hand limits may not exist!

(14) On the other hand explain how you know that

$$\lim_{x\to 0} x^2\, sin\, \frac{1}{x} = 0$$

**Hint:** Can you find functions so that $f(x) \le x^2\, sin\, \frac{1}{x} \le g(x)$ such that $f(x) \to 0$ and $g(x) \to 0$ as $x \to 0$?