

1) Compute the following triple integral

$$\int_0^1 \int_0^x \int_{-1}^{y^2} (x + 2y^2) dz dy dx$$

2) Calculate integral by changing order of integration:

$$\int_0^1 \int_y^1 x^2 e^{xy} dx dy$$

3) Show that the vector field is conservative for  $x, y > 0$  and find a potential function:

$$\vec{F}(x, y, z) = \frac{\ln y \sin z}{2\sqrt{x}} \vec{i} + \frac{\sqrt{x} \sin z}{y} \vec{j} + \sqrt{x} \ln y \cos z \vec{k}$$

4) Calculate the line integral from a parameterization

$$\int_{\gamma} xy dx + y^2 dy$$

where  $\gamma$  is parameterized as  $\vec{r}(t) = t^2 \vec{i} + \sqrt{t} \vec{j}$ ,  $1 \leq t \leq 9$ .

5) The following is an exact line integral. Compute it on the path  $\gamma$  given in cylindrical coordinates as  $r = \frac{1}{2}\theta, z = \theta$  for  $0 \leq \theta \leq 8\pi$ . Draw a graph of  $\gamma$ .

$$\int_{\gamma} 2xy \cos z dx + x^2 \cos z dy - x^2 y \sin z dz$$

6) Calculate curl and divergence of the following vector field:

$$\vec{F}(x, y) = \sin y \vec{i} + x \vec{j} + z^2 \vec{k}$$

7) Convert to cylindrical coordinates and evaluate

$$\iiint_{\mathcal{R}} \frac{z}{(x^2 + y^2 + z^2)^{\frac{5}{2}}} dV$$

where  $\mathcal{R}$  is  $\{(x, y, z) | x^2 + y^2 \leq 1, z \geq 1\}$ .

8) Write down, but do not evaluate, an expression in spherical coordinates which finds the volume of the region  $x^2 + y^2 + z^2 \leq 4$  and  $-1 \leq z \leq 1$ . You will need a sum of integrals!

9) Find the limits in the second integral

$$\int_{0, \pi/2} \int_0^{x^2} \int_0^{\cos x} f(x, y, z) dz dy dx = \int_{?}^{?} \int_{?}^{?} \int_{?}^{?} dx dy dz$$

10) Compute the following double integral:

$$\iint_{\mathcal{R}} y - x^2 dA$$

over the region  $\mathcal{R}$  defined by  $1/x \leq y \leq 8/x$  and  $x^2 \leq y \leq 3 + x^2$  by using substitutions to change the variables. (hint: put all  $x$ 's and  $y$ 's in the middle of the inequalities).