

1) Compute the following triple integral

$$\int_0^1 \int_0^x \int_{-1}^{y^2} (x + 2y^2) \, dz \, dy \, dx$$

2) Calculate integral by changing order of integration:

$$\int_0^1 \int_y^1 x^2 e^{xy} \, dx \, dy$$

3) Show that the vector field is conservative for $x, y > 0$ and find a potential function:

$$\vec{F}(x, y, z) = \frac{\ln y \sin z}{2\sqrt{x}} \vec{i} + \frac{\sqrt{x} \sin z}{y} \vec{j} + \sqrt{x} \ln y \cos z \vec{k}$$

4) Calculate the line integral from a parameterization

$$\int_{\gamma} xy \, dx + y^2 \, dy$$

where γ is parameterized as $\vec{r}(t) = t^2 \vec{i} + \sqrt{t} \vec{j}$, $1 \leq t \leq 9$.

5) The following is an exact line integral. Compute it on the path γ given in cylindrical coordinates as $r = \frac{1}{2}\theta$, $z = \theta$ for $0 \leq \theta \leq 8\pi$. Draw a graph of γ .

$$\int_{\gamma} 2xy \cos z \, dx + x^2 \cos z \, dy - x^2 y \sin z \, dz$$

6) Calculate curl and divergence of the following vector field:

$$\vec{F}(x, y) = \sin y \vec{i} + x \vec{j} + z^2 \vec{k}$$

7) Convert to cylindrical coordinates and evaluate

$$\int \int \int_{\mathcal{R}} \frac{z}{(x^2 + y^2 + z^2)^{\frac{5}{2}}} \, dV$$

where \mathcal{R} is $\{(x, y, z) | x^2 + y^2 \leq 1, z \geq 1\}$.

8) Write down, but do not evaluate, an expression in spherical coordinates which finds the volume of the region $x^2 + y^2 + z^2 \leq 4$ and $-1 \leq z \leq 1$. You will need a sum of integrals!

9) Find the limits in the second integral

$$\int_{0, \pi/2} \int_0^{x^2} \int_0^{\cos x} f(x, y, z) \, dz \, dy \, dx = \int_{?}^? \int_{?}^? \int_{?}^? dx \, dy \, dz$$

10) Compute the following double integral:

$$\int \int_{\mathcal{R}} y - x^2 \, dA$$

over the region \mathcal{R} defined by $1/x \leq y \leq 8/x$ and $x^2 \leq y \leq 3 + x^2$ by using substitutions to change the variables. (hint: put all x 's and y 's in the middle of the inequalities).