Homework for Math 152H-1 November 27

Reading: Read example 1 on pg 346, make sure you understand it! Look at table 5.3, read pages 352-353.

Homework: In the book a partition is called P, and its norm is denoted $||P||$. So, $||P|| \to 0$ means use skinnier and skinnier rectangles as the number of rectangles goes to ∞. Do pg 352 # 3, 5, 7, 55, 59, 67 and read "Approximating Finite Sums with Integrals" on pg. 392-394, do # 23, 25, 26.

As a bonus problem consider the great question from class. You have the following function:

$$f(x) = \begin{cases} 3 & x \neq \sqrt{2} \\ 1 & x = \sqrt{2} \end{cases}$$

Let's use Riemann sums to show that $\int_1^2 f(x)\,dx = 3$ (exactly as if we had $y = 3$ for our function and there was no jump at $\sqrt{2}$).

We'll do this just for Riemann sums with all rectangles having the same width ($\frac{1}{n}$). $\sqrt{2}$ will never be an endpoint of one of these sub-intervals since it is irrational. So the Riemann sum is $\frac{1}{n}\sum_{i=1}^{n} f(c_i)$ where c_i is the right endpoint of each sub-interval except when $\sqrt{2}$ is in the sub-interval. For that sub-interval we choose $c_j = \sqrt{2}$. Now, compute the value of the Riemann sum for the n-rectangles, without Σ-notation and see what happens to the result as $n \to \infty$. Why does the jump at $\sqrt{2}$ have no effect on the limit?

Adapt this argument to any partition and any choice of c_j to see that the integral does exist.

Suppose we let $f(x)$ be 3 everywhere except at a finite number of jumps, where it is defined but not equal to 3. Does $\int_a^b f(x)\,dx$ exist for any a and b?