(1)

(a) For any $\epsilon > 0$ can you find an $\delta > 0$ such that $1 < x < 1 + \delta$ implies that $\left|\dfrac{sin(x^2-1)}{x-1} - 1\right| < \epsilon$. Explain your answer using limits.

(b) For any $\epsilon > 0$ is there an $\delta > 0$ so that $|x| < \delta$ implies that $\left|x^2 sin(\frac{1}{x^3})\right| < \epsilon$?

(2) Compute the derivative of $f(x) = \sqrt{x} - \frac{1}{x^2}$ using the definition of the derivative.

(3) Without evaluating the integral, calculate the derivative of

$$F(x) = \int_{2x}^{3x} (1 - t^2)\, dt$$

where is $F(x)$ increasing? decreasing? Does it have any local maxima? Where is it concave up/down? Does it have any inflection points? What is $F(0)$? Graph this function!

(4) Compute the following integrals:

$$\int \frac{sec^2 x}{(1 + tan\, x)^2}\, dx \qquad\qquad \int_0^{\frac{\pi^2}{4}} \frac{sin\,\sqrt{x}}{3\sqrt{x}}\, dx$$

$$\int_0^8 \frac{1}{\sqrt{1 + \sqrt{1 + x}}}\, dx$$

(5) Find the tangent line to $\sqrt{x}y + x\, sin\, y = \frac{\pi}{2} + 1$ at $(1, \frac{\pi}{2})$. By how much should we change y to approximate a solution when $x = 1.01$?

(6) Find the area of the bounded region between $y = x^3 - x$ and $y = x^2 - 1$. Now compute the same area using right handed Riemann sums with equal width rectangles and a limit as $n \to \infty$.

(7) Use an integral to compute the following limit:

$$\lim_{n\to\infty} \sqrt{\frac{3}{n}} \sum_{k=1}^{n} \frac{1}{\sqrt{k}}$$

Hint: Use a function $f(x) = x^s$ with $s < 0$.

(8) Water flows into a conical filter with radius $3'$ and height $4'$. At every moment the water makes it through the filter at a rate of $2\ ft^3/hr$. The rate at which the water flows into the filter changes according to the height of the water already in the filter (denoted by h) according to $\frac{dV_{in}}{dt} = \frac{12}{(1+h)^2}$. How fast is the height changing when the water is $2'$ high in the filter? If we let the water flow in indefinitely, starting with an empty filter, will the filter ever overflow? What is the highest the water will get in the filter?

(9) Find the largest volume of a cylindrical can with radius r and height h which can be made using less that $96\pi\ ft^2$ of sheet aluminum (including both top and bottom).