MATH 461: Homework #8

1) Let (X, d) be a metric space. Prove

a) Let $x \in X$, then $X - \{x\}$ is open.

b) For any two distinct points $x, y \in X$, there are open balls $B_r(x)$ and $B_s(y)$ such that $B_r(x) \cap B_s(y) = \emptyset$

2) Suppose (X, d) is a metric space such that *every* map $f : X \to \mathbb{R}$ is continuous. Show that $\{x\}$ is an open set for every $x \in X$. What are all the open sets in X?

3) Let $f : X \to Y$ and $g : Y \to Z$ be maps of metric spaces. Use the open set definition of continuity to prove that

a) If f, g are both continuous, then $gf : X \to Z$ is continuous.

b) Suppose X can be represented as a union of open sets $\mathcal{O}_i$, $i \in I$. Show that f is continuous if and only if $f|_{\mathcal{O}_i}$ is continuous for each $i \in I$.