MATH 461: Homework #7

1) Consider a map $f : X \to (Y, d_Y)$, where only Y is equipped with a metric. Let $x_1, x_2 \in X$ and define $D(x_1, x_2) = d(f(x_1), f(x_2))$. IS (X, D) a metric space? a pseudo-metric space? Is there a condition you could place on f which will make D a metric? (think about the various set theoretic properties of functions?)

2) Show that for any $a \in (X, d)$, with d a metric, the function $f_a : X \to \mathbb{R}$ given by $f_a(x) = d(x, a)$, i.e. by measuring the distance to a, is continuous. You can use any of the definitions we have given of continuity. As a conclusion of this exercise, note that for an infinte set X, there are always loads of continuous functions!