MATH 461: Homework #26

Let $(X, \mathcal{T})$ be a T_1 space, so that points are closed. A *cut point* $p \in (X, \mathcal{T})$ is a point where $X - \{p\}$ is disconnected. Points where this does not happen are called non-cut point. For example, in the interval $[0, 1]$, every point in $(0, 1)$ is a cut point, but 0 and 1 are non-cut points.

1) Show that if $f : (X, \mathcal{X}) \to (Y, \mathcal{Y})$ is a homeomorphism, and $p \in X$ is a cut point, then $f(p) \in Y$ is also a cut point, and vice-versa.

2) Use the result of the previous exercise to show that the following are not homeomorphic:

(1) $\mathbb{R}$ and $\mathbb{R}^2$.
(2) $[0, \infty)$ and $\mathbb{R}$.
(3) $[0, 1]$ and S^1.