MATH 461: Homework #25

1) Prove that a space $(X, \mathcal{T})$ is connected if and only if there are no surjective continuous maps $f : X \to \{0, 1\}$, where $\{0, 1\}$ has the discrete topology.

2) Which of the following subspaces of $\mathbb{R}^2$ are connected? path connected?

(1) $cl(B_1(-1, 0)) \cup cl(B_1(1, 0))$
(2) $cl(B_1(-1, 0)) \cup B_1(1, 0)$
(3) the rational comb: $\left\{ (q, y) \in \mathbb{R}^2 \,\middle|\, q \in \mathbb{Q}, y \in [0, 1] \right\} \cup ([0, 1] \times \{ 1 \})$
(4) $\left\{ (x, y) \,\middle|\, x \text{ or } y \text{ is rational} \right\}$

3) Show that any infinite set, X, with the cofinite topology is connected. Is it path connected?