MATH 461: Homework #22

1) Let (X, d) be a complete metric space, and let $A \subset X$ be a subset such that $(A, d|_A)$ is also complete. Show that A must be closed in X. (Recall that for metric spaces, we can characterize when a set is closed by using sequences of points from the set).

2) Since $\mathbb{R}$ is complete, we showed in class that $C(X, \mathbb{R})$, the bounded continuous real valued functions on X, is also complete for the metric $D(f, g) = \sup_{x \in X} |f(x) - g(x)|$. Pick some point $a \in X$. For each $u \in X$ define a function $f_u(x)$ by

$$f_u(x) = d_X(x, u) - d_X(x, a)$$

(1) Show that $|f_u(x)| \leq d_X(u, a)$.
(2) Show that f_u is continuous. Hence $f_u \in C(X, \mathbb{R})$
(3) This defines a map $X \to C(X, \mathbb{R})$ by $u \to f_u$. Is this map $1 - 1$?
(4) For $u, v \in X$, what is $D(f_u, f_v)$?