MATH 461: Homework #22

1) Prove that a closed subset of a complete metric space is complete. Prove that the product of two metric spaces, $X \times Y$, with the metric $d_1(x_1, x_2) + d_2(y_1, y_2)$, is complete if and only if the two metric spaces are complete.

2) Let (X, d) be a totally bounded metric space. Let $A \subset X$ have the metric found by restricting d. Prove that A is also totally bounded.

3) Let A_i be complete subsets of a metric space (X, d) (i.e. every Cauchy sequence of elements in A_i converges to a point in A_i). Show that finite unions of these subsets are complete. Show that any intersection of these subsets is complete. (You might want to start by noting that a subsequence of a Cauchy sequence is Cauchy)