MATH 461: Homework #21

1) Prove that a compact subset of a Hausdorff space is closed. (**Hint:** For $x \in X - C$, where C is compact, consider all pairs of points $\{x, y\}$ with $y \in C$).

2) Let $(X, \mathcal{T})$ be sequentially compact. Prove that any continuous function $f : X \to \mathbb{R}$ is bounded, i.e. there exists an M such that $-M \leq f(x) \leq M$ for every $x \in X$. Use this to prove that every continuous function has a maximum.

3) Find an example of a metric space that is totally bounded, but not compact.