MATH 461: Homework #2

1) Let A be a set. Consider the map $\psi : A \to \mathcal{P}(\mathcal{P}(A))$ defined by $\psi(a) = \{B \subset A \text{ such that } a \in B\}$. For example, if $A = \{1, 2, 3\}$ then $\psi(2) = \{\{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}$.

a) Is this a function? (i.e. is this the correct range?)
b) Is this function one-to-one?
c) Is it onto?

2) We consider the following relations for elements of the real numbers, $\mathbb{R}$.

(1) $x \sim y$ if $x - y \in \mathbb{Z}$. Show that this is an equivalence relation. Is there an easy way to think about the set of equivalence classes?
(2) $x \sim y$ if $x - y \in \mathbb{Q}$ where $\mathbb{Q}$ is the rational numbers. Show that this is an equivalence relation. Is there an easy way to think about the equivalence classes? (how is this different from (1)?)
(3) $x \sim y$ if $x - y \in S$ where $S \subset \mathbb{R}$. What properties must S possess for this to define an equivalence relation?

3) Let $f : X \to Y$ be a function, and let $A_i \subset X$ for $I \in I$, and $B_j \subset Y$ for $j \in J$. Show that the following equalities are true:

$$f^{-1}(\bigcup_{j \in J} B_j) = \bigcup_{j \in J} f^{-1}(B_j)$$

$$f^{-1}(\bigcap_{j \in J} B_j) = \bigcap_{j \in J} f^{-1}(B_j)$$

$$f(\bigcup_{i \in I} A_i) = \bigcup_{i \in I} f(A_i)$$

$$f(\bigcap_{i \in I} A_i) \subset \bigcap_{i \in I} f(A_i)$$

Show, by example, that the last cannot be replaced by an equality. Again if the use of index sets bothers you, you should start by proving this when $I, J = \{1, 2\}$ (i.e. for A_1, $A_2 \subset X$ and $B_1, B_2 \subset Y$).