MATH 461: Homework #19

Note: There was a mistake on HMWK 18. The question about 2^{nd} countable should assume that the map f is open. This has been corrected on the website.

In each of the following X is a topological space, and $\sim$ is an equivalence relation on X. For $\mathbb{R}, S^1, \mathbb{R}^2$ the usual topology is presumed. q is the map from X onto the set of equivalence classes,E, and $\mathcal{Q}$ is the quotient topology of q on E. Describe $(E, \mathcal{Q})$ for each of the following equivalence relations:

1) $X = S^1$, which we will describe by angles $\theta \in [0, 2\pi)$. The equivalence classes are $\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\}$ and $\{\theta\}$ for $\theta \neq 0, \frac{2\pi}{3}, \frac{4\pi}{3}$.

2) $X = \mathbb{R}^2$ and $x \sim y$ if and only if $d(x, (0,0)) = d(y, (0,0))$ for the unsual metric.

3) $X = S^1 \times [0,1]$ with equivalence classes $\{(\theta, 1) | \theta \in S^1\}$ and $\{(\theta, r)\}$ if $r \neq 1$.

4) $X = \mathbb{R}$ and $x \sim y$ if and only if $x - y \in \mathbb{Q}$. Show that E is uncountable. Show that $\mathcal{Q} = \{\emptyset, \mathbb{R}\}$.
Hint: consider the points in the equivalence class including 0 and the open sets containing this equivalence class.

Note that $\mathbb{R}$ is T_4, but $\{\emptyset, \mathbb{R}\}$ is not even T_0. The T_i properties are not necessarily preserved under quotient maps.