**MATH 461: Homework #18**

Let $f : (X, \mathcal{T}) \to Y$ and let $\mathcal{Q}$ be the quotient topology on $Y$, i.e.

$$\mathcal{Q} = \left\{ O \subset Y \,\middle|\, f^{-1}(O) \in \mathcal{T} \right\}$$

Prove the following about $\mathcal{Q}$:

(1) $(Y, \mathcal{Q})$ *is* a topological space.
(2) If $(X, \mathcal{T})$ is separable, so is $(Y, \mathcal{Q})$.
(3) If $f$ is open and $(X, \mathcal{T})$ is $2^{nd}$ countable, so is $(Y, \mathcal{Q})$.
(4) A map $g : (Y, \mathcal{Q}) \to (Z, \mathcal{Z})$, where $\mathcal{Z}$ is a topology on $Z$, is continuous if an only if $g \circ f : (X, \mathcal{T}) \to (Z, \mathcal{Z})$ is continuous.