MATH 461: Homework #17

1) Prove the following:

(1) If $(X, \mathcal{T})$ is regular, and $S \subset X$, without restriction, then $(S, \mathcal{T}|_S)$ is also regular.

(2) Let $(Y, \mathcal{T})$ be a normal space. If $S \subset X$ is a *closed* subset, then $(S, \mathcal{T}|_S)$ is also normal. This argument is very similar to the preceding one, but there it worked for any subset. What goes wrong if you remove the restriction that S be closed?

2) Prove that the Sorgenfrey line, $\mathbb{E}$, is T_3. Recall that $\mathbb{E}$ is the space with set $\mathbb{R}$ and topology generated by $\{ [a, b) \mid a, b \in \mathbb{R} \}$. (It is T_4 in fact, but that's harder to prove.)

3) Let $\mathbb{E} \times \mathbb{E}$ be the Sorgenfrey plane.

(1) Describe a base for the open sets.

(2) What is the subspace topology on $\{ (x, y) \mid x + y = 1 \}$.

(3) Is $\{ (x, y) \mid x + y = 1 \}$ closed in $\mathbb{E} \times \mathbb{E}$?

(4) Is $\mathbb{E} \times \mathbb{E}$ separable? 1^{st} countable? 2^{nd} countable? (look back at facts about subspaces and products).

(5) Is $\mathbb{E} \times \mathbb{E}$ normal? (use the lemma from class, and the preceding questions!)