MATH 461: Homework #15

1) Suppose that X, Y are topological spaces, $f, g : X \to Y$ are continuous, and Y is Hausdorff. Show that if A is a dense subset of X and $f(a) = g(a)$ for all $a \in A$, then $f(x) = g(x)$ for all $x \in X$.

2) Let $(X_i, \mathcal{T}_i)$ be a topological space, for each $i = 1, \ldots, n$, with more than two points. Prove

a) If each X_i is second countable, so is $\prod_{i=1}^n X_i$ with the product topology.
b) If each X_i is separable, so is $\prod_{i=1}^n X_i$ with the product topology.
c) If each X_i is Hausdorff, so is $\prod_{i=1}^n X_i$ with the product topology.