MATH 461: Homework #14

1) This is the example that a subspace of a separable space, $(X, \mathcal{T})$, is not necessarily separable. The following is called the Moore Plane.

Let $X = \{(x, y) \in \mathbb{R}^2 | y \geq 0\}$ be the upper half-plane. We equip this with a new topology. Namely we use the topology generated by the elements of $\mathcal{B}$, where all the elements of $\mathcal{B}$ are given by

(1) For each (x, y) with $y > 0$, and $0 < r < y$, then $B_r(y) \in \mathcal{B}$, where B_r is the standard metric open ball.

(2) For $(x, 0)$, $B \cup \{(x, 0)\} \in \mathcal{B}$ where B is any open ball in the upper half plane *tangent* to $\mathbb{R}$ at $(x, 0)$.

Show that

a) $\mathcal{B}$ satisfies the two properties to be a basis for some topology.

b) With this topology X is separable.

c) However, $\mathbb{R} = \{y = 0\}$ as a subspace of the Moore plane is *not* separable. Fully describe the subspace topology.

2) Let $(X, \mathcal{T})$ be a separable space, and A a countable dense subset. Let $O \in \mathcal{T}$ be an open subset of X. Show that $O \cap A$ is a countable dense subset of $(O, \mathcal{T}|_O)$ where $\mathcal{T}|_O$ is the subspace topology on O.

3) Let $A \subset (X, \mathcal{T})$. Prove that

(1) If $f : (X, \mathcal{T}) \to (Y, \mathcal{T}_Y)$ is continuous, then the restriction $f|_A : (A, \mathcal{T}|_A) \to (Y, \mathcal{T}_Y)$ is continuous.

(2) If $g : (Y, \mathcal{T}_Y) \to (X, \mathcal{T})$ has image in A, there is a map $g_A : (Y, \mathcal{T}_Y) \to (A, \mathcal{T}|_A)$, found simply by changing the range. Show that g is continuous as a map from Y to X if an only if g_A is continuous as a map from Y to A.