MATH 461: Homework #12

1) Let $(X, \mathcal{T}_X)$ be a topological space. Let $A \subset X$ be any subset. Define $\mathcal{T}_A \subset \mathcal{P}(A)$ to be the collection $\{O \subset A \big| O = \mathcal{O} \cap A, \mathcal{O} \in \mathcal{T}_X\}$. Show that $\mathcal{T}_A$ is a topology on A. Note that this works for *any* subset of X. A subset A equipped with this topology is called a *subspace* of X and the topology $\mathcal{T}_A$ is called the *subspace topology*. If this seems familiar this is because in a previous homework you proved that if (X, d) is metric and $(A, d|_A)$ is the metric space found by restricting to A, then the metric topology on A is also the subspace topology.

2) Let $(X, \mathcal{T})$ have a subspace $(A, \mathcal{T}_A)$ (again, A need not be open in X). Prove

a) If $(X, \mathcal{T})$ is 1^{st} countable, so is $(A, \mathcal{T}_A)$.

b) If $(X, \mathcal{T})$ is 2^{nd} countable, so is $(A, \mathcal{T}_A)$.

c) If $(X, \mathcal{T})$ is Hausdorff, so is $(A, \mathcal{T}_A)$. (Recall: Hausdorff (also: T_2) is the property that given any two distinct points x, y in X, there are *disjoint* open sets, O_1, O_2, such that $x \in O_1$ and $y \in O_2$.)

3) Let $\mathcal{S}$ be the set of all the lines in $\mathbb{R}^2$, not necessarily through the origin. Describe the topology on $\mathbb{R}^2$ for which $\mathcal{S}$ is a sub-basis.