MATH 461: Homework #12

1) Let X be a set. Let $\mathcal{T}_i \subset \mathcal{P}(X)$ be a topology on X for each $i \in I$. Show that $\bigcap_{i\in I} \mathcal{T}_i \subset \mathcal{P}(X)$ is also a topology.

2) Let X be $\mathbb{R} \cup \{\alpha\}$, where α is some other element of the set, not a real number. Let $\mathcal{B}$ consist of the open intervals (a, b) in $\mathbb{R}$, and the sets $(a, 0) \cup \{\alpha\} \cup (0, b)$ for $a < 0 < b$. In other words, the elements of $\mathcal{B}$ containing α are the intervals containing 0, but with 0 replaced by α. Show that $\mathcal{B}$ is a basis and that in this topology, any sequence converging to 0 also converges to α. (This is sometimes called "duplicating a point").

3) Let $X = \mathbb{R}$ and let $\mathcal{B} = \{[a, b) \big| a, b \in \mathbb{R}\}$ be the set of half open intervals. Show

(1) $\mathcal{B}$ satisfies the two properties of a basis, and thus $\mathcal{T}_{\mathcal{B}}$ is a topology on $\mathbb{R}$.
(2) The collection $[x, x + \frac{1}{n}), n \in \mathbb{N}$ is a neighborhood basis for x, so $\mathcal{T}_{\mathcal{B}}$ is first countable.
(3) $[a, b)$ is also closed, for each $a, b \in \mathbb{R}$.
(4) Points are closed, and not open (these are *not* the same thing!).

$\mathbb{R}$ with this topology is called the *Sorgenfrey* line.