MATH 461: Homework #11

First, in problem #2 of the previous homework (#10), choose $r_i < \frac{M}{4}$, not "over two" as stated in the problem. The point of this condition is to exclude a radius large enough that you can include all of X_i in the ball. This is a tricky problem, so here's a hint: show that the set in the problem cannot contain a ball of radius ϵ about $\{x_i\}$ by examining what points are in the ball for large i. Also, the result is not true for finite products (it *is* a neighborhood there), so something about this being an infinite product must play a role.

1) Recall that $\mathcal{T} \subset \mathcal{P}(X)$ is a topology on a set X if

i) $X, \emptyset \in \mathcal{T}$

ii) If $O_i \in \mathcal{T}$ for every $i \in I$ then $\bigcup_{i \in I} O_i \in \mathcal{T}$

iii) If $O_1, \ldots, O_n \in \mathcal{T}$ then $O_1 \cap O_2 \cap \cdots \cap O_n \in \mathcal{T}$.

Show that the following are all topologies:

(1) Any set X with $\mathcal{T} = \{X, \emptyset\}$ (the indiscrete topology)
(2) Any set X with $\mathcal{T} = \mathcal{P}(X)$ (the discrete topology)
(3) If $f : X \to (Y, \mathcal{S})$, with $\mathcal{S}$ a topology on Y, then $\mathcal{T} = \{f^{-1}(O) | O \in \mathcal{S}\}$ is a topology on X.