MATH 461: Final part #1

Name:

Answer all questions with as much detail as possible. Answers without justification will not receive credit. Insufficient justification will lead to points being taken away. You may consult books and other references, however you may not consult other people besides me. No credit will be given unless you sign the statement of academic integrity below and return it with you answers.

I, ________________________, S.I.D. ______, performed my own work in the completion of this portion of my exam. I did not ask or receive any assistance from any person other then the instructor. I wrote my solutions myself and did not have them checked or reviewed by any other person. I have maintained the standards on plagarism incorporated in Michigan State's policy on academic honesty.

Signature:____

1) Show that if $(X, \mathcal{T})$ is a compact, Hausdorff space, no finer topology on X will be compact and no coarser topology will be Hausdorff. (There's a very nice result about continuous bijections from a compact space to a Hausdorff space, which you can find in Sutherland, which is useful here once you find the right map!)

2) Let $I = [0, 1] \subset \mathbb{R}$. Suppose there are closed intervals $C_1, \ldots, C_n \subset I$ such that $C_i \cap C_j = \emptyset$ if $i \neq j$. Let $x \sim y$ if $x, y \in C_i$ for some i. If $x \notin C_i$ for all i then $x \sim x$ only. Prove that this is an equivalence relation. If the set of equivalence classes is provided with the quotient topology, what topological space results? (Give as much justification as you can muster.)

3) Say that two metric spaces, (X, d_X) and (Y, d_Y) are uniformly equivalent if there is a bijection $f : X \to Y$ for which f and f^{-1} are both uniformly continuous. Show that X is complete if and only if Y is complete.

4) Let $X = \mathbb{N}$ as a set.

a) Show that the sets $\emptyset, X$ and $\{1, \ldots, n\}$ for $n \in \mathbb{N}$ provide a topology to X.

b) Prove that X with this topology is not compact.

c) Prove that every continuous map $f : X \to \mathbb{R}$ is constant (hence bounded).

d) What are the components of this space?

5) For each $n \in \mathbb{N}$, let C_n be a compact connected subspace of a Hausdorff space, $(X, \mathcal{T})$. Suppose $C_{n+1} \subset C_n$ for all n. Prove that $\bigcap_{n\in\mathbb{N}} C_n$ is connected.